• Exercise : Lecture 4 •

"The Higgs as a Moderator"

The aim of this exercise is to make contact among the SO(4) σ-model and the Higgs model for Electro-Weak Symmetry Breaking

A) Given the Higgs field H, transforming as a doublet of $SO(2)_L$ and with $Y=1/2$ hypercharge, define out of it a matrix

$$\Phi = (H, H^c) \quad ; \; H^c = (-i\sigma_2) H^*$$

$\Phi=(i\sigma_2)\Phi^*(i\sigma_2)$ ✓

show that Φ fullfills the pseudo-reality condition so that it can be expanded as

$$\Phi = h_4 \mathbb{1} + i h_i \sigma^i$$

Find the h_m components explicitely in terms of the components of H. Show that the SO(4) linear σ-model Lagrangian for h_m coincides, after expressing h_m in terms of H, with the usual SM Higgs Lagrangian. This shows that the two theories coincide!

B) Knowing how $SU(2)_L$ acts on H, how it acts on the Φ matrix? What about the hypercharge? Even though it was not easy to see this when looking to the Higgs Lagrangian written with the H doublet, the Higgs model has a larger group than $SU(2)_L \times U(1)_Y$, which group?

c) In the "non-linear" basis (of $\sigma, \vec{\pi}$ fields), the massive physical Higgs boson is described by the σ field, while the $\vec{\pi}$ are the "would-be" Goldstone bosons that are normally eaten by the $W^\pm$ and Z vector bosons. The $\vec{\pi}$'s are, in a sense, "part" of the $W^\pm, Z$ vector fields. They provide, basically, their Longitudinal Polarization.

Compute the $\pi\pi \to \pi\pi$ scattering amplitude in the limit of very high energy: $E \gg m_\sigma$. The contribution from the direct 4-π vertex discussed in the lesson will grow with the energy. The other contributions, coming from virtual σ exchange, will cancel this growth and make the amplitude constant.

There is a rigorous sense (the Equivalence Theorem) in which what you have computed is the high-energy scattering amplitude of longitudinally polarized vector bosons. The role of the σ (Higgs) particle is to make weaker the force among vector bosons at very high energy. This is what your calculation has shown