Exercises: Lecture 1

1) Take N real scalars:

$$\phi_i \; ; \quad i=1,\dots,N$$

with Lagrangian

$$\mathcal{L} = \frac{1}{2}\sum_{i=1}^{N} \partial_\mu \phi_i \partial^\mu \phi_i + \frac{\mu^2}{2}\sum_i \phi_i^2 - \frac{g}{4}\left(\sum_i \phi_i^2\right)^2$$

Questions:

A) What symmetries (Space-time, Global and Discrete) this theory has?

B) At the classical level, what is the vacuum configuration and which symmetries are spontaneously broken?

C) In the simple case $N=2$, what is the spectrum of fluctuations around the vacuum? Check that the Goldstone Theorem holds

D) What happens for generic N?

2) Be G a Lie group of symmetries, of generators $\{t^A\}$. If the operator Φ_i takes a VEV, the generators can be split in un-broken and broken generators: $\{t^A\} = \{t^a, t^{\hat{a}}\}$. Show that the $\{t^a\}$ form a subalgebra of the total $\mathcal{L}ie[G]$ algebra.