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Lectures on AFT, EPFL

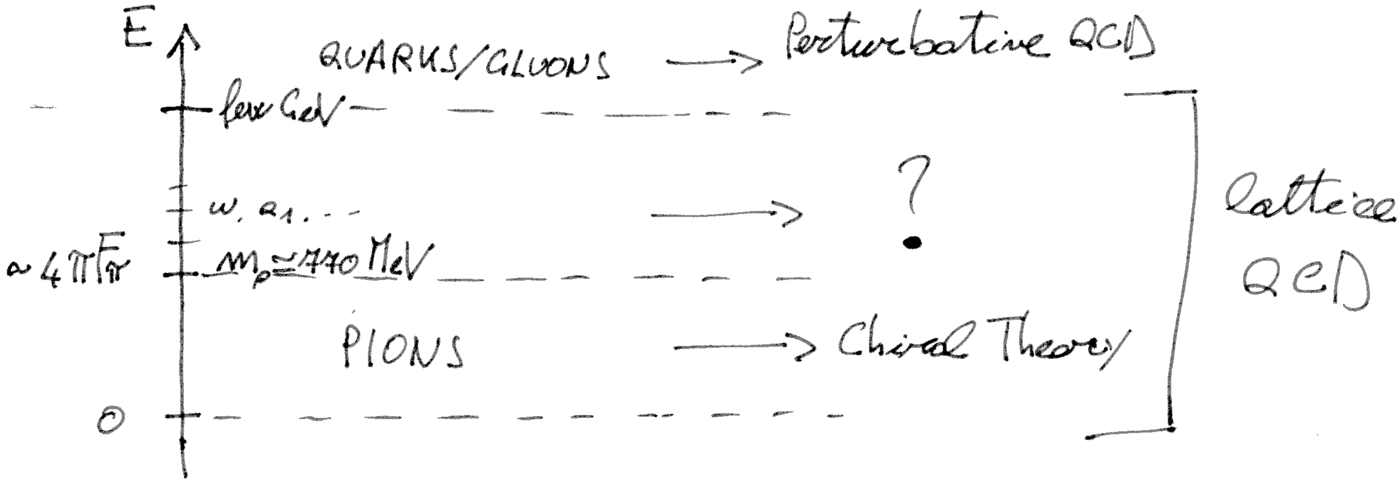
On Effective Field Theories.

The logic followed until now is inspired by the historical development of the modern approach to Effective Field Theories (EFT). We first noticed how useful the chiral effective theory is for the description of the observed light mesons, and therefore got interested in theories that are rather different from the "standard", renormalizable, ones. This led us to a change of perspective on the role of Renormalizability, and pushed us towards a different approach to any Quantum Field Theory (QFT):

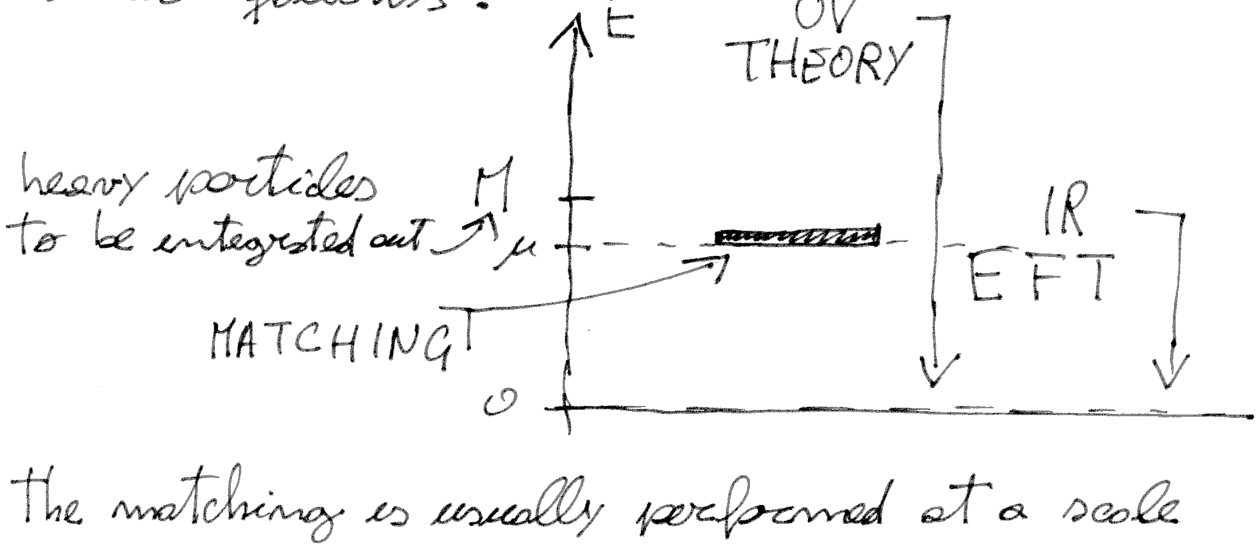
QFT $\equiv$ "Given Set of Fields, or better of PARTICLES,
+ SYMMETRIES"

The Lagrangian is a derived object, containing all the possible local operators compatible with the Principles. These ideas are very much related to the one of EFT, as we will now discuss. Consider for example a QFT which contains a particle of mass M, plus other particles of mass $m \ll M$, and imagine to perform low-energy experiments, at $m \lesssim E \ll M$. The heavy particle has no chance of being produced; the low-energy experiments only consist of light particle scatterings. These scatterings obviously respect all the symmetry of the underlying theory, but will never contain the heavy particle as a physical propagating state. For what these low-energy experiments are concerned, therefore, it is like if the theory contained one particle less, but the same symmetries as the original one. If particles and symmetries is all what matters, shouldn't our low-energy Physics be equally well described by a new theory, a low-energy EFT, in which the heavy particle is not present at all from the beginning while the symmetries of the original theory are manteined? This is the general idea behind the use of the EFT: given that two theories with the same content of <u>visible particles</u> and of symmetries <u>must coincide</u>, I can always construct <u>simplified</u> models in which only the relevant degrees of freedom are retained. The non-relevant ones, i.e. the ones which are too heavy to be produced at the energies we are interested in, are said to be "integrated out" from the original theory. The previous statement, that two theories with same particle content and symmetries must coincide, is not based on a theorem. It is a "conjecture" based on the idea that a QFT with a totally generic local Lagrangian merely leads to the most general result for the scattering amplitudes, compatible with the symmetries (Lorentz, in the first place) and with causality.

These considerations lead to a much more transparent motivation for building the Chiral Theory of pions. The Goldstone theorem is just needed to ensure that the 3 massless (or light) pions exist, and to read their quantum number under the chiral group. Once we know that, and assume that there are not in the theory other (unnaturally) light particles, the above considerations imply that it must be enough to build a model in which only the 3 light states are retained. Driven by symmetries, this model will be constructed as we discussed in the last lesson; it will contain all the terms allowed by the symmetries. At low energies (we saw last time that "low" means $E/4\pi F_\pi < 1$) the theory is predictive and, since it contains the same particles and symmetries, it <u>coincides with QCD</u>. In a picture, the status of the phenomenology of the QCD strong force is as follows:



When we say that the effective theory coincides with the original (or, say, "fundamental") one, it is of course understood that the <u>parameters</u> of the two theories must have been choosen in an appropriate way. When possible, or useful, the parameters (i.e., the renormalized couplings) can be chosen as usual by comparing with the experiments. This can be done for the EFT (as we did for F_π in the chiral theory) <u>and</u> for the fundamental one, separately. This was not possible in the case of QCD because we were in the non-perturbative regime. Another option, when experiment are not available or in other situations, is to match the parameters of the EFT in terms of those of the fundamental theory. In this case, one basically employs the fundamental theory as if it was the experimental input. The renormalization conditions are given, in the process of renormalization of the EFT, not by the measured value of the scattering amplitudes but by the predictions for these amplitudes one obtains in the fundamental theory. The scheme of such matching procedure is as follows:



the matching is usually performed at a scale $\mu \lesssim M$. This typically avoids large logarithms.

The program above can obviously be completed only if the fundamental theory is also calculable in the region of energy where the EFT holds. The usefulness of the EFT approach, however, may appear questionable in this case. The chiral theory is so useful because it is <u>the only handle</u> on QCD in the region of the far IR, this is so of course only because QCD is not calculable at that energies. If the fundamental theory is calculable, why not to use it directly? There are several reasons:

1) The use of EFT simplifyies the calculations

2) The use of EFT makes <u>transparent</u> the physical origin of the phenomenon one is willing to study. This is essential to understand where and how a certain physical principle can be tested.

EXAMPLE: how much the existence of the W bosons affects the muon decay?

3) Interpreting the theories we know as Effective can help in guessing how the more Fundamental Theory should look like, more on this later.

Let us discuss a simple example of this matching procedure. Our "true", fundamental, theory is just

$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} m^2 \varphi^2 + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \\ - \frac{1}{2} M^2 \phi^2 - \frac{1}{2} \kappa \varphi^2 \phi$$

where we suppose that $\kappa \sim m << M$. This theory merely describes 2 real scalars, one of mass m, light, one of mass M, heavy. Let us regard this theory, which is renormalizable, as a genuinely fundamental theory valid up to arbitrarily high energies (be aware that this theory has plenty of problems, among which the one of vacuum stability, which however will not affect our discussion). Notice the presence of a Z_2 reflection symmetry $\varphi \to -\varphi$, under which the heavy field is even, $\phi \to \phi$.

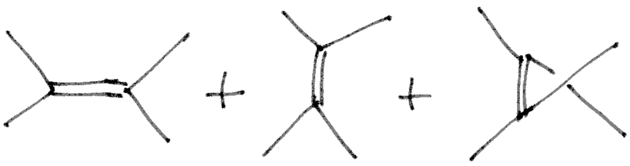
At $E<M$, where the ϕ particle cannot be produced, we can use our effective theory, which describes a "light" real scalar $\bar{\varphi}$, endowed with a Z_2 $\bar{\varphi} \to -\bar{\varphi}$ symmetry, and therefore with the most

general allowed Lagrangian

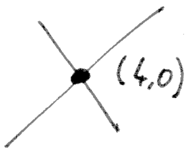
$$\mathcal{L} = \frac{1}{2}\partial_\mu \bar{\varphi}\,\partial^\mu \bar{\varphi} - \frac{1}{2}\bar{m}^2 \varphi^2 - \frac{1}{4!} C^{(4,0)} \varphi^4 + - \frac{1}{3!} C^{(4,2)} \varphi^3 \Box \varphi \quad + \dots$$

where the coefficients are labeled in terms of the number of fields and of derivatives of the associated operator. Notice that we have changed name to the φ field, calling it $\bar{\varphi}$, with the aim of stressing that the fundamental and effective theories are described by <u>completely unrelated set of fields</u>, which <u>must be thought to live in two separate field spaces</u>. Think to QCD and to the pions!

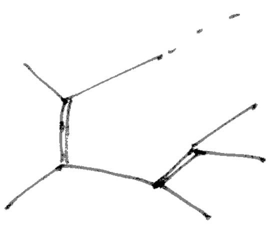
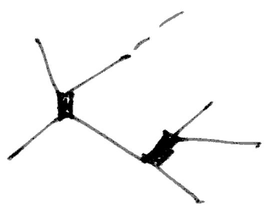
The true and the effective theories will coincide once we will have fixed the parameters. As explainded before, let us match the parameters of the EFT to the fundamental one. To start with, let us work at the tree-level and match the quadrilinear coupling $C^{(4,0)}$. This is most easily done by computing the 4-point function at zero external momentum. In the true theory this gives:

 $= -3\kappa^2 \dfrac{i}{0 - M^2}$

while in the EFT one has

 $= -i\, C_0^{(4,0)} \Rightarrow C_0^{(4,0)} = -\dfrac{3\kappa^2}{M^2} \ll 1$

The quadrilinear coupling is small, showing that the EFT will be a perturbative one. For what concerns the mass $\bar{m}$, it clearly equals m at the tree-level. With the two parameters $\bar{m}$ and $C_0^{(4,0)}$, the EFT is completely specified for what concerns its leading order predictions, in an E^2/M^2 expansion. It is also immediate to check that the EFT exactly reproduces the fundamental one at this order. Consider indeed a generic amplitude of momenta $q_i \sim E$:

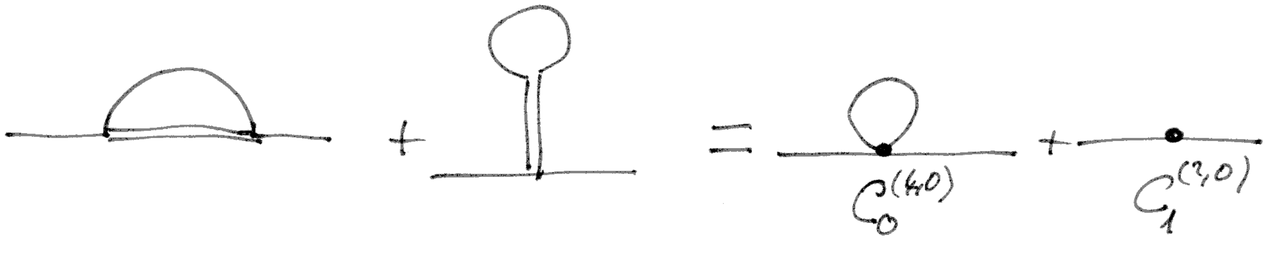
 $=$  $+ \mathcal{O}(E^2/M^2)$

the tree-level meaning of the EFT method is trivial: just shrink the propagators to give contact interactions. The effect of the microphysics is merely of fixing the coefficient of this interaction term.

The situation becomes more involved if willing to include E^2/M^2 corrections. As we know, these come from both the higher dimensional operators such as $C^{(4,2)}$ and from loops of the lower dimensional ones [$C^{(4,0)}$, in the present case]. One should also remember that the (renormalized) value of the couplings must be corrected if willing to reach higher precision. We will therefore write

$$C^{(i,j)} = C_0^{(i,j)} + C_1^{(i,j)} + \cdots$$

An example of matching at one loop order is as follows. Compute the 2-point function at zero momentum in both theories, working in the $\overline{MS}$ renormalization scheme. The diagrams in the fundamental and in the EFT are



and must be equated for the two theories to match. Notice that the tree-level part of the 2-point function has already been matched by imposing $\overline{m}^2 = C_0^{(3,0)} = m^2$.

Also, notice that in the one-loop diagram it is sufficient to employ the tree-level coupling $C_0^{(4,0)}$, and there is no need for its one-loop correction. You will show as an exercise that this matching condition reads

$$i\frac{K^2}{(4\pi)^2}\left(1-\frac{M^2}{M^2-m^2}\log\left(\frac{M^2}{\mu^2}\right)+\frac{m^2}{M^2-m^2}\log\frac{m^2}{\mu^2}\right)$$
$$-\frac{i}{2}\frac{K^2}{(4\pi)^2}\frac{m^2}{M^2}\left(1-\log\frac{m^2}{\mu^2}\right)=\frac{i}{2}\,\frac{C_0^{(4,0)}}{(4\pi)^2}m^2\left(1-\log\frac{m^2}{\mu^2}\right)+$$
$$-iC_1^{(2,0)}$$

You know that large logarithms signal a breaking of the "naive" perturbation theory and need, if present, to be resummed by some Renormalization Group Equation method. It is much simpler, however, to avoid these logs by suitably choosing the renormalization scale μ. Taking $\mu=M$ one obtains:

$$i\frac{K^2}{(4\pi)^2}\left(1-\frac{1}{2}\frac{m^2}{M^2}+\frac{3}{2}\frac{m^2}{M^2}\log\frac{m^2}{M^2}+\mathcal{O}\left(\frac{m^4}{M^4}\right)\right)=$$
$$=\frac{i}{2}\frac{C_0^{(4,0)}}{(4\pi)^2}m^2\left(1-\log\frac{m^2}{M^2}\right)-iC_1^{(2,0)}$$

Notice that both sides of the equality contain a large log. However, for $C_0^{(4,0)} = -3K^2/M$, i.e. its correct value as obtained from tree-level matching, the log cancels! This little miracle is due to the fact that the $\log \frac{m^2}{M^2}$ is related to the running from the renormalization scale $\mu = M$ to the low scale m. But in this range of energy the true and the effective theories coincide, so they lead to the same running corrections. Our matching condition then becomes:

$$C_1^{(2,0)} = -\frac{K^2}{16\pi^2}$$

At the same order where $C_1^{(2,0)}$ is needed, we also need higher derivatives tree-level vertices, such as $C^{(4,2)}$. This comes from subleading terms in the Taylor expansion of the $2 \to 2$ amplitude. You will compute it in an exercise.