(Weinberg 19.1)

• Spontaneous Symmetry Breaking. ①

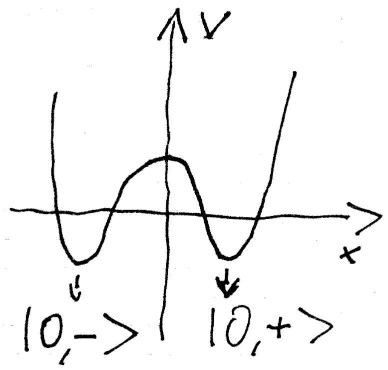
In QFT, symmetry of the Hamiltonian might not result in a symmetry of the particle spectrum.

The particles might not form multiplets of the symmetry, and this is only possible if the vacuum is not invariant under the symmetry. If $|0\rangle$ was invariant $\boxed{\text{i.e., if } Q|0\rangle = 0}$

$$M = \langle 0| Q_{\vec{0}} H Q^{+}_{\vec{0}} |0\rangle = \langle 0| e^{i\alpha_i Q^i} Q_{\vec{0}} e^{-i\alpha_i Q^i} H e^{i\alpha_i Q^i} Q^{+}_{\vec{0}} e^{-i\alpha_i Q^i} |0\rangle$$

I can rotate each $Q^i_{\vec{0}}$ into $Q^{\alpha}_{\vec{0}} = e^{i\alpha Q} Q_{\vec{0}} e^{-i\alpha Q}$, I have a family of degenerate states that form a linear representation of the symmetry group.

Spontaneous Symmetry Breaking does not occur in Quantum Mechanics:



If tunneling is taken into account:

$$\mathcal{H} = \begin{pmatrix} \langle +|H|+\rangle & \langle +|H|-\rangle \\ \langle -|H|+\rangle & \langle -|H|-\rangle \end{pmatrix} = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$$

$$|0,\text{True}\rangle = \frac{1}{\sqrt{2}}\left[\,|+\rangle \pm |-\rangle\,\right]$$

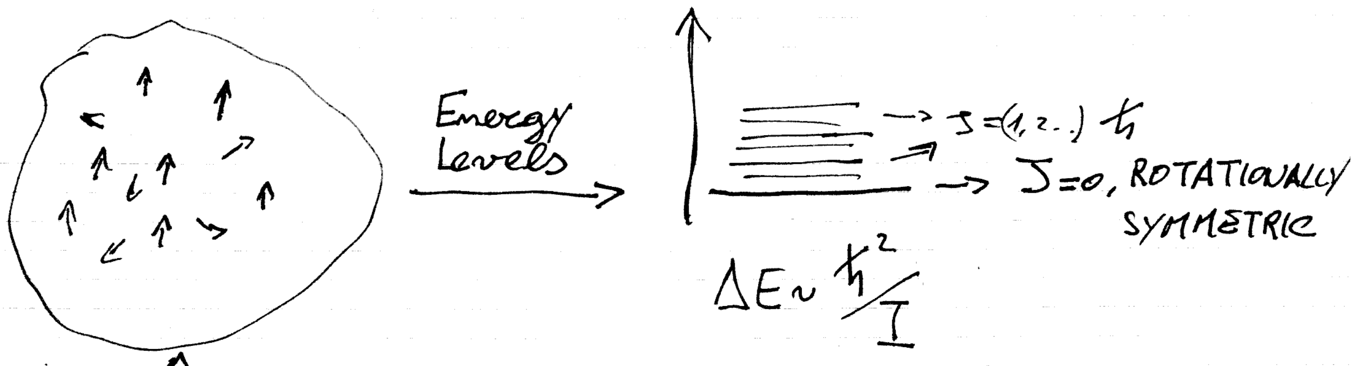
→ depending on sign (b)

$$E_{\text{True}} = (a \pm b)$$

$|0, \text{True}\rangle$ is <u>invariant</u> (up to a phase)

Similarly, for a continuous symmetry the $|0\rangle$ is typically invariant (s-wave, in the case of rotation symmetry)

However, consider a <u>very</u> <u>large</u> <u>system</u>:



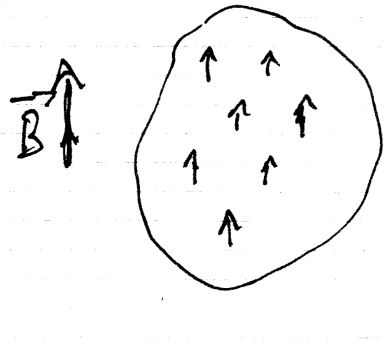
$$\Delta E \sim \hbar^2/I$$

spins, or a chair

I = "macroscopic moment of inertia"

As Volume $\to \infty$, $I \to \infty$, $\delta E \to 0$

Any tiny perturbation changes vacuum radically:



As $V \to \infty$, $B \to 0$, <u>this configuration</u> is <u>stable</u> (only one path over many makes all spins rotate simultaneously)

Theorem:

At $V \to \infty$, tunneling $b \sim e^{-cV} \longrightarrow 0$

Proof: $|v\rangle$ = "vacuum of a QFT in ∞ Volume"

$$\vec{P} \cdot |v\rangle = 0$$

Be $|u\rangle, |v\rangle$ a (discrete) set of vacua

EQUAL-TIMES $\quad \langle u | v \rangle = \delta_{uv}$

$$\langle u | A(\vec{x})\, B(0) | v \rangle = \sum_w \langle u | A(0) | w \rangle \langle w | B(0) | v \rangle + \int d^3 p \, \langle u | A(0) | \vec{p} \rangle \langle \vec{p} | B(0) | v \rangle \cdot e^{-i \vec{p} \cdot \vec{x}}.$$

$$\Big\downarrow \; |\vec{x}| \to \infty \quad \text{(Riemann-Lebesgue)}$$

$$\sum_w \langle u | A(0) | w \rangle \langle w | B(0) | v \rangle$$

Causality: $\quad [A(\vec{x},0), B(\vec{0},0)] = 0 \quad (\vec{x} \neq \vec{0})$

$\Rightarrow \; A_{uw} \equiv \langle u | A(0) | w \rangle$ commutes with all others:

$\forall A: \quad \langle u | A(0) | v \rangle = a_v \delta_{uv}$ [By changing Basis]

$\Rightarrow \; \langle u | H | v \rangle = E_\mu \delta_{uv}$: Hamiltonian diagonal

$\langle u | H' | v \rangle = E'_\mu \delta_{uv}$: Even Symmetry-breaking Perturbation is diagonal

This implies:

(1) No $|u\rangle \to |v\rangle$ transitions by H, symmetry then makes the two degenerate

(2) No $|u\rangle \to |v\rangle$ from <u>any</u> perturbation H' either (made of local operators) Perturbation just splits their energies

The above theorem shows that the tunneling among different vacua does not occur at infinite volume, so that spontaneous symmetry breaking becomes possible. Let us then consider a theory with spontaneous breaking; by definition this means that

$$Q^A|0\rangle \neq 0 \qquad \text{for some } A$$

where Q^A is the generator of the symmetry transformation. A generic local field $\phi_n(x)$ forms a representation of the symmetry group which means

$$[Q^A, \phi_n(x)] = t^A_{nm}\, \phi_m(x)$$

where t^A are the generators in the appropriate (5)
representation. Let us compute

$$\langle 0|[Q^A, \phi_n(0)]|0\rangle = t^A_{nm}\langle 0|\phi_m(0)|0\rangle$$

$$\shortparallel$$

$$\langle 0|Q^A\phi_n(0)|0\rangle - \langle 0|\phi_n(0)Q^A|0\rangle$$

Now, first suppose that the symmetry (or at least the generator "A") is unbroken; we have that any local operator ϕ_n must have a neutral VEV (Vacuum Expectation Value):

$$t^A_{nm}\langle 0|\phi_m(0)|0\rangle = 0$$

Therefore, only neutral fields can take a VEV if the symmetry is unbroken.

If, instead, the symmetry is broken, $Q^A|0\rangle \neq 0$ and therefore

$$t^A_{nm}\langle 0|\phi_m|0\rangle \neq 0$$

$$\phi_{vac,n} \equiv \langle 0|\phi_n|0\rangle$$

The VEV of fields that are charged under the symmetry (i.e. $t^A \neq 0$) is now different from zero. More precisely, the statement is that at least one operator $\exists$ which is charged under the symmetry and has non-vanishing VEV

The converse is also true: If $\langle \phi_n \rangle \neq 0$ we see from the formula above that $Q^A |0\rangle$ must be different from zero. The existence of an "order parameter" i.e. of a local operator ϕ that takes VEV is the necessary and sufficient condition for Spontaneous Symmetry breaking to occur.