Quantum Field Theory II
Exercise sheet 8

Exercise 8.1 Coleman-Weinberg potential in the $\lambda\phi^4$ theory

In the lectures you saw that the effective potential at one loop can be written as

$$V_{1-LOOP} = i \sum_{n=1}^{\infty} \frac{1}{2n} \int \frac{d^d k}{(2\pi)^d} \left(\frac{\lambda \phi^2}{2} \right)^n \frac{1}{(k^2 - m^2)^n}.$$

a) Integrate on the loop momentum k and introduce the Pochhammer symbol

$$(\alpha, \beta) = \Gamma(\alpha + \beta)/\Gamma(\alpha)$$

to recast the above expression into the form

$$V_{1-LOOP} = -\frac{1}{2} \left(\frac{m^2}{4\pi} \right)^{d/2} \Gamma(-d/2) \left(\sum_{n=1}^{\infty} \frac{(-d/2, n)}{n!} w^n \right),$$

where w has to be determined; then use the relation $\sum_{n=0}^{\infty} \frac{(-d/2,n)}{n!} w^n = (1-w)^{d/2}$ (note that the sum starts from $n = 0$ in this case), substitute $d = 4 - 2\epsilon$, expand in ϵ and note how many terms diverge as $1/\epsilon$.

b) Compute the 2- and the 4-point functions up to one loop for zero external momenta.

Now let's indroduce mass and coupling constant counterterms

$$m^2 = m_R^2 (1 + \delta_m),$$

$$\lambda = \lambda_R (1 + \delta_\lambda).$$

Find expressions for these counterterms, requiring them to cancel the one loop diagrams exactly.

c) Write the full effective potential up to one loop and show that the counterterms you found above cancel indeed the divergencies.