
Exercise 7.1 Bosonic propagators

In the lecture you have learned how to derive the propagator for massless gauge bosons in R_ξ gauges, which are defined by the gauge condition $G(A) - w(x)$ with $G(A) = \partial_\mu A^{a,\mu}$ and by multiplying the generating functional Z by a factor $C = \int \mathcal{D}w \exp(-i \int d^4x \frac{w(x)^2}{2\xi}$. This is equivalent to adding a term $-\frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2$ to the Lagrangian.
In this exercise you will consider two different cases. In a) you will treat a massive vector boson while in b) you will impose a different gauge condition, called the axial gauge.

a) Find the inverse of the operator:

$$\left[\left(-\partial^2 + M^2\right) g_{\mu\nu} + \left(1 - \frac{1}{\xi}\right) \partial_\mu \partial_\nu\right] .$$

This will be the case of a massive gauge-boson such as W, Z.

b) Find the gauge boson propagator in an axial gauge

$$G(A) = \eta_\mu A^{\mu a},$$

where η is a light-like vector ($\eta^2 = 0$). Proceed analogously to the derivation of the bosonic propagator as you have seen in the lecture and set $\xi = 1$.

Hint: Go to momentum space and choose a suitable Ansatz for the propagator.
The propagator must fulfill

$$\Delta^{ab}_{F,\mu\nu} \left(\Delta^{(-1)}_F\right)^{\nu\lambda,bc} = \delta^{ac} \delta^\lambda_\mu .$$

Exercise 7.2 A three-point function in QCD

Calculate the three-point function $\langle 0|T A^a_\mu(x_1) \bar{\psi}_i(x_2) \psi_j(x_3)|0\rangle$ to first order in the coupling strength g, using the path integral formalism.

Hints:

- Start from eq (271) on p. 63
- Use your knowledge from ex. 3.4 and 4.1.
- You may want to think ahead and only calculate those pieces that won't vanish when setting the source terms to zero.

Exercise 7.3 BRST Jacobian

Show that the path integral is invariant under BRST transformations, i.e. show that the Jacobian of the transformation is unity.

a) Start with a 2 dimensional integral $I = \int dy d\eta\, f(y, \eta)$, where y is a bosonic degree of freedom while η is fermionic. Consider a transformation

$$y = x + \lambda a(x, \xi) \qquad \eta = \xi + \lambda b(x, \xi) \ ,$$

where λ is Grassmannian, i.e. $\lambda^2 = 0$.
Rewrite I in terms of the new variables x (bosonic) and ξ (fermionic). What is the Jacobian?
Hints:

- Write the Jacobian as $J = 1 + \lambda j(x, \xi)$.
- Integrate partially and use the identity

$$\int d\xi \frac{df(\xi)}{d\xi} g(\xi) = \mp \int d\xi \frac{dg(\xi)}{d\xi} f(\xi) \ ,$$

where the sign is $-$ if f is commuting and $+$ if it is anti-commuting. ξ is anti-commuting.

b)* Now derive the Jacobian for the measure $\mathcal{D}A_\mu^a \mathcal{D}\eta \mathcal{D}\bar{\eta} \mathcal{D}\psi \mathcal{D}\bar{\psi}$ under a BRST transformation and show that it is unity.
Hints:

- Write down the transformation matrix for a BRST transformation and use a generalization of a) to derive the Jacobian.
- You might want to solve this exercise using properties of supermatrices (see exercise sheet 5).