Advanced Field Theory
Solution 3

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Exercise 3.1 Pseudoscalar Higgs Coupling to Gluons

Note: The numerical prefactor of $\langle |\mathcal{M}|^2 \rangle$ is incorrect. See for example [1] for the correct prefactor.
We start by evaluating the trace in

$$\begin{aligned} \mathcal{M}_1^{\alpha\beta} = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \left(\frac{-i}{\sqrt{2}} y_f \delta_{lj}\right) \left(-i g_s T^a_{ji}\right) \left(-i g_s T^b_{il}\right) \\ \cdot \operatorname{Tr}\left(\frac{(\gamma^5)(i)(\not{p}_1 + \not{k} + m_f)\gamma^\alpha (i)(\not{k} + m_f)\gamma^\beta (i)(\not{k} - \not{p}_2 + m_f)}{\left((p_1+k)^2 - m_f^2\right)\left(k^2 - m_f^2\right)\left((k-p_2)^2 - m_f^2\right)} \right) \end{aligned} \tag{1}$$

since the integral is finite, we set $d = 4$ from the start, we arrive at

$$\operatorname{Tr}\left(\gamma^5 \left((\not{k} + \not{p}_1 + m)\gamma^\alpha(\not{k} + m_f)\gamma^\beta(\not{k} - \not{p}_2 + m)\right)\right)$$

only traces with γ^5 and four other gamma matrices are nonvanishing

$$= m_f \operatorname{Tr}\left(\gamma^5 \left((\not{k} + \not{p}_1)\gamma^\alpha \not{k}\gamma^\beta + (\not{k} + \not{p}_1)\gamma^\alpha\gamma^\beta(\not{k} - \not{p}_2) + \gamma^\alpha \not{k}\gamma^\beta(\not{k} - \not{p}_2)\right)\right)$$

expand, terms with two $\not{k}$ vanish

$$= m_f \operatorname{Tr}\left(\gamma^5 (\not{p}_1\gamma^\alpha \not{k}\gamma^\beta + \not{p}_1\gamma^\alpha\gamma^\beta\not{k} - \not{p}_1\gamma^\alpha\gamma^\beta\not{p}_2 - \not{k}\gamma^\alpha\gamma^\beta\not{p}_2 - \gamma^\alpha\not{k}\gamma^\beta\not{p}_2)\right)$$

using antisymmetry under interchange of two adjacent gamma matrices

$$= 4 i m_f \epsilon^{\mu\nu\alpha\beta}(p_1)_\mu (p_2)_\nu .$$

Inserting this into (1) together with $\operatorname{Tr}(T^a T^b) = 1/2\ \delta^{ab}$ we have

$$\mathcal{M}_1^{\alpha\beta} = \frac{y_f g_s^2}{\sqrt{2}} \frac{\delta^{ab}}{2} (4 i m_f)\epsilon^{\mu\nu\alpha\beta}(p_1)_\mu (p_2)_\nu \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{\left[(p_1+k)^2 - m_f^2\right]\left[k^2 - m_f^2\right]\left[(k-p_2)^2 - m_f^2\right]} .$$

We will denote the remaining integral as $I(p_1, -k_2)$ in the following. The second diagram is obtained from the first one via $p_1 \to p_2$, $\alpha \to \beta$ and $k \to -k$, therefore we have the same contribution as from the first diagram. In total, we have

$$\mathcal{M}_{1+2}^{\alpha\beta} = \frac{1}{\sqrt{2}} y_f g_s^2 \delta^{ab} (4 i m_f) \epsilon^{\mu\nu\alpha\beta}(p_1)_\mu (p_2)_\nu I(p_1, -k_2).$$

Proceeding to the summation over external polarisations and colour indices, we have

$$\left\langle |\mathcal{M}|^2 \right\rangle = \frac{1}{N_p^2 N_g^2} \sum_{\text{polarisations}} (\epsilon_1^*)_\nu (\epsilon_1)_\mu (\epsilon_2^*)_\rho (\epsilon_2^*)_\sigma \left(\mathcal{M}_{1+2}\right)^{\nu\rho} \left(\mathcal{M}_{1+2}^*\right)^{\mu\sigma} .$$

In the present case, we can replace $\sum_{\text{polarisations}} (\epsilon_1^*)_\mu (\epsilon_1)_\nu \to -g_{\mu\nu}$ as in QED. Using $\epsilon^{\alpha\beta\delta\xi}\epsilon_{\mu\nu\delta\xi} p_1^\alpha p_2^\beta p_1^\mu p_2^\nu = 2(p_1\cdot p_2)^2$ (due to $p_i^2 = 0$) and $\delta^{aa} = N_g$ we have

$$\left\langle |\mathcal{M}|^2 \right\rangle = \frac{1}{N_p^2 N_g} 8\, y_f^2 g_s^4 m_f^2 2 (p_1\cdot p_2)^2 \left| I(p_1, -p_2)\right|^2 .$$

We can insert $N_p = 2$, $N_g = 8$, $g_s^2 = 4\pi\alpha_s$, $p_1 \cdot p_2 = m_h^2/2$, $y_f = \sqrt{2}m_f/v$ where v is the vacuum expectation value of the Higgs. In this form we have

$$\left\langle |\mathcal{M}|^2 \right\rangle = \frac{1}{4}\frac{m_f^4}{v^2}(4\pi\alpha_s)^2 m_h^4 \left|I(p_1, -p_2)\right|^2 .$$

We can go from the matrix element (the averaging over initial state spins and colours will be implicit from now) to the partonic cross section using

$$\mathrm{d}\hat{\sigma}_{g\,g\to h} = \frac{1}{2s_{gg}} \left|\mathcal{M}\right|^2 (2\pi)^4 \delta^{(4)}(p_{g_1} + p_{g_2} - p_h) \int \frac{\mathrm{d}^3 p_h}{(2\pi)^3)} \frac{1}{2E_h}$$

from which we get to the integrated partonic cross section, inserting $\int \mathrm{d}^3 p \frac{1}{2E}\big|_{\mathbf{p}^2+E^2=m^2} = \int \mathrm{d}^4 p\, \delta(p^2 - m^2)$:

$$\hat{\sigma}_{g\,g\to h} = \frac{\pi}{m_h^2}\delta(s_{gg} - m_h^2) \left|\mathcal{M}\right|^2 .$$

The cross section for $p\,p \to h$ is given by

$$\begin{aligned}\sigma_{p\,p\to h} &= \int \mathrm{d}x_1 \mathrm{d}x_2 g(x_1) g(x_2) \hat{\sigma}_{g\,g\to h} \\ &= \int \mathrm{d}x_1 \mathrm{d}x_2 g(x_1) g(x_2) \frac{\pi}{m_h^2}\delta(s_{gg} - m_h^2) \left|\mathcal{M}\right|^2 .\end{aligned}$$

where g denotes the gluon parton distribution function of the proton.
We can execute one of the integrals in the convolution:

$$\int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 g(x_1) g(x_2) \delta(s_{gg} - m_h^2) = \frac{1}{s}\int_{\frac{m_h^2}{s}}^1 \mathrm{d}x_1 g(x_1) g\left(\frac{m_h^2}{s}\frac{1}{x_1}\right) .$$

Putting it together, we have

$$\sigma_{p\,p\to h} = \frac{\pi}{m_h^2}\left|\mathcal{M}\right|^2 \frac{1}{s}\int_{\frac{m_h^2}{s}}^1 \mathrm{d}x_1 g(x_1) g\left(\frac{m_h^2}{s}\frac{1}{x_1}\right) .$$

We insert the matrix element into this formula together with $I(p_1, -p_2) = \frac{-i}{(4\pi)^2}\frac{1}{m_h^2}\frac{1}{2}f(\tau)$ to have

$$\sigma_{p\,p\to h} = \frac{1}{16}\frac{1}{16\pi}\frac{m_f^4}{v^2 m_h^2}\alpha_s^2 \left|f(\tau)\right|^2 \left(\frac{1}{s}\int_{\frac{m_h^2}{s}}^1 \mathrm{d}x_1 g(x_1) g\left(\frac{m_h^2}{s}\frac{1}{x_1}\right)\right) .$$

A detailed derivation for the production of a scalar can be found in Mathias Bruchereifers Masters Thesis, it can be found at `http://www.itp.phys.ethz.ch/education/lectures_fs10/AFT_FS_10`.

References

[1] M. Spira, A. Djouadi, D. Graudenz and P. M. Zerwas, Phys. Lett. B **318** (1993) 347.