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Advanced Field Theory
Assignment 10

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Exercise 10.1 The soliton of $\lambda\phi^4$ theory in $1+1$ dimensions

Consider the scalar field theory with Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi), \qquad V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

in one dimension ($g_{\mu\nu} = \mathrm{diag}(+1, -1)$) with $m^2 < 0$.

1. Determine the constant solutions of the equation of motion, shift the potential such that these have vanishing energy.

2. Find the static solutions of the equations of motion interpolating between two constant solutions (these interpolating solutions are called solitons), use the ansatz $\phi(x) = a\tanh(bx)$.

3. Calculate the energy of the static soliton ($\int \mathrm{d}x \cosh^{-4}(x) = 4/3$)

4. Check that the current
$$J^\mu = \epsilon^{\mu\nu}\partial_\nu\phi$$
is conserved. What are the possible values of $\int J^0 \mathrm{d}x$ for a solution of the equation of motion?