ETH
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Advanced Field Theory
Assignment 4

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Exercise 4.1 Weak Decay of the Pion

1. Consider the Lagrangian for semileptonic weak interactions:

$$\mathcal{L} = \frac{4G_F}{\sqrt{2}} \left(\bar{\ell}_L \gamma^\mu \nu_L\right) \left(\bar{u}_L \gamma_\mu d_L\right) + \text{h.c.}$$

with $\nu_L = P_L \nu = 1/2(1-\gamma^5)\nu$.
Using the quark currents defined as

$$J^{\mu a} = \bar{Q}\gamma^\mu \tau^a Q \qquad J^{\mu 5a} = \bar{Q}\gamma^\mu \gamma^5 \tau^a Q$$

where $Q = (u\ d)^T$ is the quark doublet and $\tau^a = \sigma^a/2$ are the generators of $SU(2)$, show that

$$\bar{u}_L \gamma^\mu d_L = \frac{1}{2}\left(J^{\mu 1} + iJ^{\mu 2} - J^{\mu 51} - iJ^{\mu 52}\right).$$

2. The matrix element of $J^{\mu 5a}$ between the vacuum and an on-shell pion can be written as

$$\langle 0|J^{\mu 5a}|\pi^b(p)\rangle = -ip^\mu f_\pi \delta^{ab} e^{-ip\cdot x}$$

where f_π is a constant with the dimension of a mass. Using this identification together with the result of part 1, show that the amplitude for the decay $\pi^+ \to \ell^+\nu$ ($|\pi^+\rangle = 1/\sqrt{2}(|\pi^1\rangle + i\,|\pi^2\rangle)$) is given by

$$i\mathcal{M} = G_F f_\pi \bar{u}(q)\not{p}(1-\gamma^5)v(k)$$

where p, k and q are the momenta of the π^+, ℓ^+ and ν respectively.

3. Compute the decay rate of the pion. Show that this rate vanishes in the limit of zero lepton mass, and that the relative rate of pion decay to muons and electrons is given by

$$\frac{\Gamma(\pi^+ \to e^+\nu)}{\Gamma(\pi^+ \to \mu^+\nu)} = \left(\frac{m_e}{m_\mu}\right)^2 \frac{\left(1 - m_e^2/m_\pi^2\right)^2}{\left(1 - m_\mu^2/m_\pi^2\right)^2} = 10^{-4}.$$

From the measured pion lifetime $\tau_\pi = 2.6 \cdot 10^{-8}$ s, $G_F = 1.17 \cdot 10^{-5}$ GeV^{-2}, $1\,\text{s} = 1.52 \cdot 10^{21}$ MeV^{-1} and the pion and muon masses $m_\pi = 140$ MeV, $m_\mu = 106$ MeV, determine the value of f_π.