Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Advanced Field Theory
Assignment 1

Spring 2010
C. Anastasiou
T. Gehrmann

Exercise 1.1 Dimensional Regularisation

Prove the following identities:

$$\int \frac{\mathrm{d}^d p_E}{(2\pi)^d} \frac{1}{(p_E^2+\Delta)^n} = \frac{1}{(4\pi)^{d/2}} \frac{\Gamma\left(n-\frac{d}{2}\right)}{\Gamma(n)} \Delta^{\frac{d}{2}-n}$$
$$\int \frac{\mathrm{d}^d p_E}{(2\pi)^d} \frac{p_E^2}{(p_E^2+\Delta)^n} = \frac{1}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma\left(n-\frac{d}{2}-1\right)}{\Gamma(n)} \Delta^{1+\frac{d}{2}-n}.$$

You will need to make use of the integral representation of the beta function

$$\int_0^1 \mathrm{d}x x^a (1-x)^b = B(a+1,b+1) = \frac{\Gamma(a+1)\Gamma(b+1)}{\Gamma(a+b+2)}$$

and of the integral over the surface of the d-sphere

$$\int \mathrm{d}^{d-1}\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}.$$

Exercise 1.2 Feynman Parameters

Prove the formula for the Feynman parametrisation of n propagators by induction:

$$\frac{1}{D_1^{a_1}\dots D_n^{a_n}} = \frac{\Gamma(a_1+a_2+\dots+a_n)}{\Gamma(a_1)\Gamma(a_2)\dots\Gamma(a_n)} \times \int_0^1 \mathrm{d}x_1 \dots \int_0^1 \mathrm{d}x_n \frac{\delta(1-x_1-x_2-\dots-x_n)\, x_1^{a_1-1} x_2^{a_2-1} \dots x_n^{a_n-1}}{[x_1 D_1+\dots+x_n D_n]^{a_1+\dots+a_n}}.$$

One way verify the identity for $n=2$ is by direct integration, for this one can use the hypergeometric integral

$$\int_0^1 \mathrm{d}x x^a (1-x)^b (1-z\,x)^c = \frac{\Gamma(a+1)\Gamma(b+1)}{\Gamma(a+b+2)}\, {}_2F_1(-c,a+1,a+b+2;z)$$
$$= \frac{\Gamma(a+1)\Gamma(b+1)}{\Gamma(a+b+2)} \sum_{k=0}^{\infty} \frac{\Gamma(-c+k)}{\Gamma(-c)} \frac{\Gamma(a+1+k)}{\Gamma(a+1)} \frac{\Gamma(a+b+2)}{\Gamma(a+b+2+k)} \frac{z^k}{k!}$$

Exercise 1.3 Generalisation of γ_5 in d dimensions

In four dimensions, γ_5 can be defined by its two properties:

$$(1) \qquad \mathrm{Tr}\,(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5) = \epsilon_{\mu\nu\rho\sigma}\,\mathrm{Tr}\,\mathbf{1}$$
$$(2) \qquad \{\gamma_\mu,\gamma_5\} = 0$$

Show that those two properties cannot be maintained simultaneously in d dimensions, by showing that (2) leads to

$$(d-2)(d-4)\,\mathrm{Tr}\,(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5) = 0$$