Quantum Mechanics II
Exercise 5

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Exercise 5.1 Slow Turn-On of Perturbation

Consider a harmonic oscillator in the eigenstate $|i\rangle$. At time $t = 0$, it is subjected to a time-dependent perturbation $V(t)$

$$V(t) = Ve^{i\omega t} + V^\dagger e^{-i\omega t}, \tag{1}$$

where V and $V^\dagger$ are time-independent operators (not necessarily hermitian).

a) Calculate the probability and rate of the system for the transition to state $|f\rangle$ after a time t, where $i \neq f$.

b) Now let's turn the perturbation on slowly by modifying the potential slightly.

$$V(t) = (Ve^{i\omega t} + V^\dagger e^{-i\omega t})e^{\delta t}. \tag{2}$$

Assume that in the remote past, $t \to -\infty$, the oscillator is in state $|i\rangle$. Compute the transition probability and transition rate of the system for the transition to state $|f\rangle$ at some time in the future t, where $i \neq f$.

Exercise 5.2 Sudden Constant Perturbation

Consider a system with the Hamiltonian

$$H = H_0 + \Theta(t)\, V, \tag{3}$$

where V is a time-independent operator of the perturbation, with

$$H_0|i\rangle = |i\rangle \quad \text{for the state } |i\rangle, \tag{4}$$

and

$$V|f\rangle = \lambda|i\rangle, \quad \forall\, |f\rangle \neq |i\rangle. \tag{5}$$

The states $|f\rangle$ have a probability density

$$\rho\left(E_f\right) = \frac{1}{\pi}\frac{1}{1+E_f^2}. \tag{6}$$

a) Show that $\int_{-\infty}^{+\infty} \rho\left(E_f\right) dE_f = 1$.

b) Compute the transition probability for arbitrary time t.

c) When does the *Golden Rule* become a good approximation?

Exercise 5.3 The Delta Function

Prove that

$$\lim_{x\to\infty} \frac{\sin\left[x\left(\omega - \omega_0\right)\right]}{\omega - \omega_0} = \pi\delta\left(\omega - \omega_0\right). \tag{7}$$