Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Quantum Information Theory
Serie 5

FS 09
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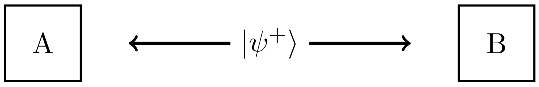
Exercise 5.1 Quantum operations can only decrease distance

Given a trace-preserving quantum operation $\mathcal{E}$ and two states ρ and σ, show that

$$\delta\left(\mathcal{E}(\sigma), \mathcal{E}(\rho)\right) \leq \delta(\sigma, \rho). \tag{1}$$

What physical principle implies that this statement has to hold?
Hint: Use Exercise 4.1.

Exercise 5.2 Bell-type Experiment



Consider a two dimensional Hilbert Space $\mathcal{H}$ with basis $\{v_1, v_2\}$, and the Bell-state $\psi^+ \in \mathcal{H} \otimes \mathcal{H}$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}\left(|v_1\rangle|v_1\rangle + |v_2\rangle|v_2\rangle\right). \tag{2}$$

Assume that, after a system with state $|\psi^+\rangle$ has been created at some time t_0, the two parts separate and propagate towards opposite observers A and B. Observer A then performs a measurement $\mathcal{M}_A^\alpha = \{|\alpha\rangle\langle\alpha|, |\alpha\rangle^\perp\langle\alpha|^\perp\}$, with $|\alpha\rangle := \cos(\alpha)|v_1\rangle + \sin(\alpha)|v_2\rangle$, on his part of the system.

a) Give the expressions for the partial state at B after the measurement at A, depending on whether the result of the measurement at A is known or not.

b) Determine the probability distribution of a measurement $\mathcal{M}_B^0$ at B conditioned/not conditioned on the measurement $\mathcal{M}_A^\alpha$ at A.

c) Convince yourself with a) and b) that the subjective assignment of states at B does not contradict the objective measurement results of B.

Exercise 5.3 Stabilizers

This exercise introduces the important formalism of stabilizers, which often allows for a more efficient representation of quantum codes and errors in cryptographic applications.
The Pauli-Group G_n is the smallest closed group which contains all possible n-fold tensor products of the Pauli-matrices $\mathbb{1}, X, Y, Z$. Let S be a subgroup of G_n and let $\mathcal{H}$ be an n-qubit Hilbert space. We say that an element $|\phi\rangle \in \mathcal{H}$ is stabilized by an operator $O \in S$ if $O|\phi\rangle = |\phi\rangle$. We define $V_S \subseteq \mathcal{H}$ to be the set of states which are stabilized by all elements of S.

a) Which necessary conditions does S have to fulfill such that V_S is non-trivial?

b) Show that V_S is the intersection of the subspaces fixed by each operator in S, and that V_S is a subspace itself.

c) Show that $|\psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, where $\{|0\rangle, |1\rangle\}$ is the computational basis, is stabilized by $X_1 \otimes X_2$ and $Z_1 \otimes Z_2$. Find a state that is stabilized by $S = \{X \otimes Z, Z \otimes X\}$?