Quantum Field Theory II, Exercise Set 11.

FS 08/09

A Slavnov–Taylor Identity

Slavnov–Taylor identities are derived using the BRST symmetry of gauge-fixed path integrals. This yields a general master formula for the effective action of the theory. In this exercise we consider a simple argument which yields a specific Slavnov–Taylor identity. Our starting point is the following, non-trivial, statement: Since the BRST transformation is an exact symmetry of the theory, all Green's functions of the theory are BRST invariant. This statemant holds for adequately regularized theory, as well as for renormalized theories. For simplicity we assume that the theory is regularized in a gauge invariant way, in this case the BRST transformation is given by

$$\begin{aligned}
\delta_{\mathrm{BRST}}\,\psi &= -g\mathrm{i}t^a\Theta c^a\psi\,,\\
\delta_{\mathrm{BRST}}\,\bar\psi &= \mathrm{i}g\bar\psi t^a\Theta c^a\,,\\
\delta_{\mathrm{BRST}}\,A^a_\mu &= \Theta(\partial_\mu c^a + gf^{abc}c^bA^c_\mu)\,,\\
\delta_{\mathrm{BRST}}\,c^a &= \frac{g}{2}f^{abc}c^bc^c\Theta\,,\\
\delta_{\mathrm{BRST}}\,c^{*\,a} &= -\frac{1}{a}\Theta\partial_\mu A^{\mu\,a}\,,
\end{aligned}$$

where the Grassmann variable Θ parametrizes the transformation. The Faddeev–Popov and gauge fixing Lagrangians are given by

$$\begin{aligned}
\mathcal{L}_{\mathrm{ghost}} &= c^{*\,a}[-\delta^{ab}\Box - gf^{abc}\partial^\mu A^c_\mu]c^b\,,\\
\mathcal{L}_{\mathrm{gauge}} &= -\frac{1}{2a}(\partial^\mu A^a_\mu)^2\,.
\end{aligned}$$

a) Starting from the BRST invariance of the Green's function $\langle 0|\mathrm{T}\,A^a_\mu(x)c^{*\,b}(y)|0\rangle$ show that

$$\langle 0|\mathrm{T}(\partial_\mu c^a(x) + gf^{adb}c^d(x)A^c_\mu(x))c^{*\,b}(y)|0\rangle = \frac{1}{a}\langle 0|\mathrm{T}A^a_\mu(x)\partial_\nu A^{\nu\,b}(y)|0\rangle\,. \tag{1}$$

b) Differentiate equation (1) with respect to x^μ. Then use the equation

$$\Box_x\langle 0|\mathrm{T}\,c^a(x)c^{*\,b}(y)|0\rangle + gf^{adc}\partial^\mu_x\langle 0|\mathrm{T}\,(c^d(x)A^c_\mu(x))c^{*\,b}(y)|0\rangle + \mathrm{i}\delta^{ab}\delta(x-y) = 0 \tag{2}$$

to derive the following Slavnov–Taylor identity

$$\frac{1}{a}\partial^\mu_x\partial^\nu_y\langle 0|\mathrm{T}A^a_\mu(x)\,A^b_\nu(y)|0\rangle = -\mathrm{i}\delta(x-y)\delta^{ab}\,. \tag{3}$$

c) What is the physical content of (3)?

Hint: Take the Fourier transform.

d)* Derive equation (2).