The finite temperature transition in QCD

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Outline

Thermodynamics

- Grand canonical ensemble is used to describe QCD. This ensemble has a fixed temperature T and a fixed chemical potential μ.
- The density matrix of the grand canonical ensemble is

$$\rho = \exp\left(-\beta\left(H - \sum_i \mu_i N_i\right)\right).$$

The partition function is

$$Z = \text{Tr}(\rho) = \sum_i \langle i |\rho| i\rangle .$$

Thermodynamics

- The thermodynamical quantities are defined as

$$
\begin{aligned}
P &= \frac{\partial(T \ln Z)}{\partial V} \\
S &= \frac{\partial(T \ln Z)}{\partial T} \\
E &= TS - PV + N_i \mu_i \\
N_i &= \frac{\partial(T \ln Z)}{\partial \mu_i} \\
F &= -PV + N_i \mu_i \\
\Omega &= E - TS - N_i \mu_i = -PV
\end{aligned}
$$

Examples of phases diagrams

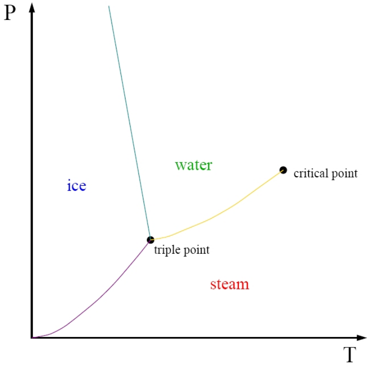


Figure: The phase diagram of water.

Examples of phases diagrams

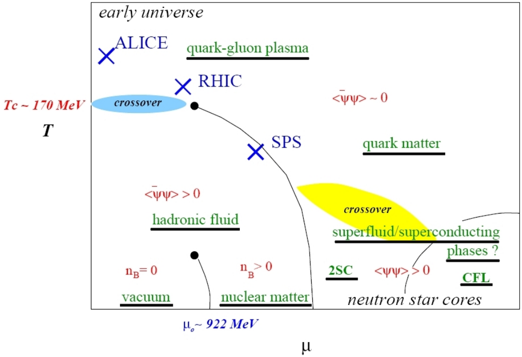


Figure: Proposed phase diagram for QCD. 2SC and CFL refer to diquark condensate, SPS, RHIC and ALICE are the names of experiments with heavy-ions collision.

Examples of phases diagrams

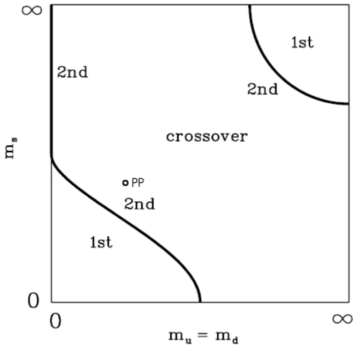


Figure: A possible phase diagram for QCD: Columbia Plot.

Outline

Facts of QCD

- The coupling constant of strong interactions vanishes for small separation $\longrightarrow$ Asymptotic freedom.
- The potential between a quark and an antiquark increases linearly for big separation $\longrightarrow$ Confinement.
- The Bag model is a simple model that takes into account this two properties of QCD.

Bag model

- Assume that there are two different vacuums:
  - the trivial vacuum with
$$F(\text{trivial vacuum}) = 0,$$
  - the true vacuum with
$$F(\text{true vacuum}) = -\Lambda_B^4.$$
- Assume that an hadron replaces R^3 true vacuum with trivial vacuum.

$$F(\text{hadron}) = -\Lambda_B^4 V + \Lambda_B^4 (V - R^3) = \Lambda_B^4 R^3$$

Bag model

- The energy of an hadron is then

$$E \approx R^3 \Lambda_B^4 + \frac{C}{R}.$$

- It follows

$$M \approx 4 R^3 \Lambda_B^4.$$

We can calculate an approximate value of the Bag constant using the values of a nucleon ($M \approx 1000$ MeV, $R \approx 1$ fm).

$$\Lambda_B \approx 200 \text{ MeV}$$

Bag model $0 < T < T_c$

- We have that $m_u \approx 2.5$ MeV, $m_d \approx 5$ MeV and $m_\pi \approx 140$ MeV
- We assume that this masses are almost zero (good for $T > 100$ MeV)
  Hence chiral symmetry holds $\rightarrow$ Spontaneous breaking $\rightarrow$ Pions (pseudo Goldstone bosons).
- We assume $B = 0$ and the pressure of the pions gas is

$$P_\pi = -\frac{(\partial T \ \ln Z)}{\partial V}\bigg|_{T,\mu} = 3 \times \left(\frac{\pi^2}{90}\right) T^4.$$

Bag model $T > T_c$

- Assume that the system behaves like $T \to \infty$ for all temperatures $T > T_c$. Hence the quarks and the gluons can move freely because of asymptotic freedom.
- The pressure of the quark gluon plasma is

$$P_{q\bar{q}} + P_g = 2 \times 2 \times 3 \times \frac{7}{4} \times \left(\frac{\pi^2}{90}\right) T^4 + 2 \times 8 \times \left(\frac{\pi^2}{90}\right) T^4.$$

The critical temperature of the Bag model

- Assume that two different states are in equilibrium if they have the same pressure
- Considering the two different vacuums, it follows that

$$\frac{1}{30}\pi^2 T_c^4 = \frac{37}{90}\pi^2 - \Lambda_B^4.$$

- The critical temperature is $T_c \approx 144$ MeV $\approx 2 \cdot 10^{12}$ K.

The equations of state of the Bag model

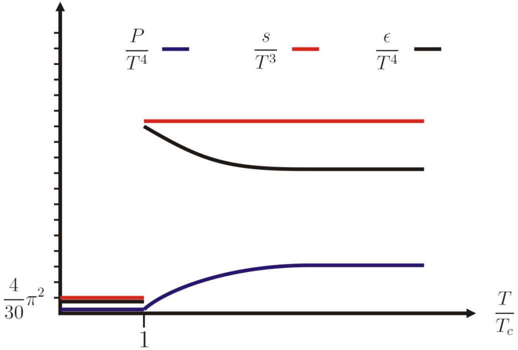


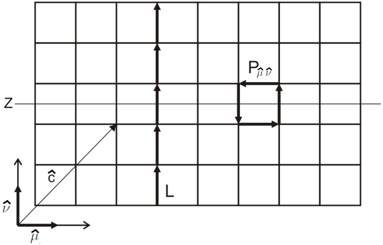
Figure: The equations of state of Bag model versus T/T_c.

Outline

Center symmetry

- The transformation of the loop $P_{\hat{\mu}\hat{\nu}}$ under a center transformation is

$$P_{\hat{\mu}\hat{\nu}} \to \left(U_{\hat{\mu}}(\hat{m}) Z U_{\hat{\mu}}(\hat{m}+\hat{\mu}) U^{\dagger}_{\hat{\mu}}(\hat{m}+\hat{\nu}) U^{\dagger}_{\hat{\mu}}(\hat{m}) Z^{\dagger} \right).$$



Polyakov loop

- The Polyakov loop is invariant under gauge transformations.
- Under center symmetry it transforms in the fundamental representation

$$L \longrightarrow ZL.$$

- It is used as order parameter, and there is a phase transition (first order for SU(3) pure gauge theory)

Non-abelian Wilson action of lattice QCD

- We recall that the non-abelian Wilson action of lattice QCD is

$$
\begin{aligned}
S &= (\hat{m} + 4r) \sum_{n} \bar{\psi}(n)\psi(n) \\
&\quad - \frac{1}{2} \sum_{n,\mu} \Big(\bar{\psi}(n)(r - \gamma_\mu) U_\mu(n) \psi(n+\mu) \\
&\qquad + \bar{\psi}(n+\mu)(r + \gamma_\mu) U_\mu^\dagger(n) \psi(n) \Big) \\
&\quad + \frac{2}{g^2} \, \mathrm{Tr} \sum_{n,\mu<\nu} \left[\mathbf{1}_3 - \frac{1}{2}\left(P_{\mu\nu}(n) + P_{\mu\nu}^\dagger(n)\right) \right],
\end{aligned}
$$

Fermions in pure gauge theory

- Allow quarks to be in the lattice.
- The action, under an application of a center transformation, will transform

$$\bar{\psi}(\hat{m})(r-\gamma_{\hat{\mu}})U_{\hat{\nu}(\hat{m})}\psi(\hat{m}+\hat{\nu})+\bar{\psi}(\hat{m}+\hat{\nu})(r+\gamma_{\hat{\nu}})U^{\dagger}_{\hat{\nu}}(\hat{m})\psi(\hat{m}) \rightarrow$$

$$\bar{\psi}(\hat{m})(r-\gamma_{\hat{\mu}})ZU_{\hat{\nu}}(\hat{m})\psi(\hat{m}+\hat{\nu})+\bar{\psi}(\hat{m}+\hat{\nu})(r+\gamma_{\hat{\nu}})U^{\dagger}_{\hat{\nu}}(\hat{m})Z^{\dagger}\psi(\hat{m}).$$

$\rightarrow$ The center symmetry is broken.

Pure gauge theory lattice simulations

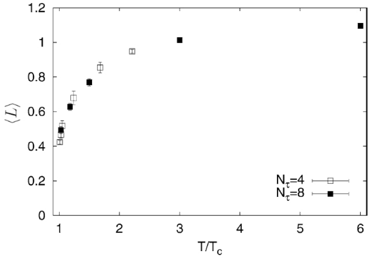


Figure: The expectation value of the Polyakov loop as a function of the temperature. The spatial lattice size in $N_s = 32^3$. The expectation value is zero below the critical temperature T_c.

Pure gauge theory lattice simulations

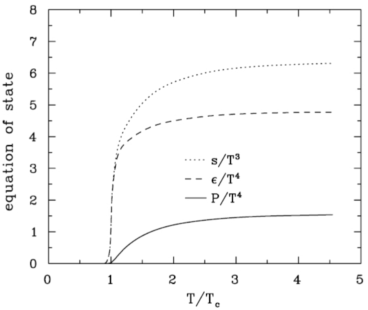


Figure: The equation of state of the pure SU(3) gauge theory versus T/T_c.

Outline

Chiral symmetry

- The Lagrangian with N_F flavours f of massless fermions is

$$\mathcal{L}^0 = \sum_f [\bar{q}_{f,L}(i\not{D})q_{f,L} + \bar{q}_{f,R}(i\not{D})q_{f,R}] - \frac{1}{4}\mathcal{G}_{a,\mu\nu}\mathcal{G}_a^{\mu\nu},$$

- It is invariant under the following transformation

$$\text{Vec: } \begin{pmatrix} f_{1\,R,L} \\ \vdots \\ f_{N_F\,R,L} \end{pmatrix} \longrightarrow e^{-i\theta_V} \begin{pmatrix} f_{1\,R,L} \\ \vdots \\ f_{N_F\,R,L} \end{pmatrix}$$

$$\text{Ax: } \begin{pmatrix} f_{1\,R,L} \\ \vdots \\ f_{N_F\,R,L} \end{pmatrix} \longrightarrow e^{-i\theta_A\gamma^5} \begin{pmatrix} f_{1\,R,L} \\ \vdots \\ f_{N_F\,R,L} \end{pmatrix}.$$

$\longrightarrow$ SU$(N_F)_V\times$ SU$(N_F)_A$ Symmetry.

Definition of the chiral condensate

- The chiral condensate is defined as

$$\langle \bar{\psi}\psi \rangle = \langle \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \rangle \,,$$

- Transformation under a vectorial chiral transformation F_V:

$$\langle \bar{\psi}\psi \rangle \rightarrow \left\langle \bar{\psi}_L F_V^\dagger F_V \psi_R + \bar{\psi}_R F_V^\dagger F_V \psi_L \right\rangle = \langle \bar{\psi}\psi \rangle \,,$$

- whereas under an axial transformation $e^{-i\theta_A \gamma^5}$:

$$\langle \bar{\psi}\psi \rangle \rightarrow \left\langle \bar{\psi}_L e^{-i\theta_A} e^{-i\theta_A} \psi_R + \bar{\psi}_R e^{i\theta_A} e^{i\theta_A} \psi_L \right\rangle \,.$$

- It is used as order parameter, and there is a phase transition.

Type of phase transition

- The universality argument.
- $SU(N_F)_A$ spontaneusly broken $\rightarrow N_F^2 - 1$ Goldstone modes.
- Consider the system of $\Phi \in N_F \times N_F$ matrices that transform as

$$\Phi \rightarrow L \Phi R.$$

- The most general (super-) renormalizable Lagrangian invariant under symmetries of this system is:

$$\begin{aligned} \mathcal{L}_\Phi &= c_o \mathrm{Tr}\left(\partial_\mu \Phi^\dagger \partial_\mu \Phi\right) - c_1 \mathrm{Tr}\left(\Phi^\dagger \Phi\right) - c_2 \left(\mathrm{Tr}\left(\Phi^\dagger \Phi\right)\right)^2 \\ & -c_3 \mathrm{Tr}\left(\Phi^\dagger \Phi\right)^2 - c_4 \mathrm{Re}\left(\det \Phi\right). \end{aligned}$$

Type of phase transition

- Assume that $\Phi = \lambda U$
- The potential is then

$$V = c_1 |\lambda|^2 + (c_2 + c_3) |\lambda|^4 + c_4 \mathrm{Re}\left(\lambda^{N_F}\right).$$

- For $N_F = 2$

$$V = a |\lambda|^2 + b\mathrm{Re}\left(\lambda^2\right) + c |\lambda|^4$$

$\longrightarrow$ second order phase transition.

- For $N_F = 3$

$$V = a |\lambda|^2 + b\mathrm{Re}\left(\lambda^3\right) + c |\lambda|^4$$

$\longrightarrow$ first order phase transition.

Mass term

- The mass term of the Lagrangian is

$$\mathcal{L}^M = \sum_f \left(\bar{f}_L + \bar{f}_R\right) m_f \left(f_L + f_R\right).$$

Under an axial transformation $f_R \to A f_R$ and $f_L \to A^\dagger f_L$, $A \in \mathrm{SU}(n)$, the mass term transforms as

$$\mathcal{L}^M \to \sum_f \left(\bar{f}_L A + \bar{f}_R A^\dagger\right) m_f \left(A^\dagger f_L + A f_R\right).$$

$\to \mathrm{SU}(N_F)_A$ chiral symmetry is broken

Chiral condensate lattice simulations

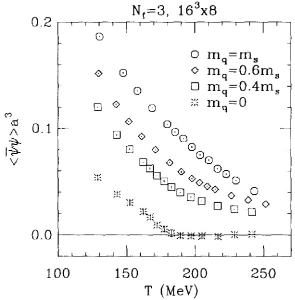


Figure: The chiral condensate $\langle \bar{\psi}\psi \rangle$ measured in lattice units in function of the temperature. The quarks mass are assumed to be the same for the up, down and strange flavours.

Chiral condensate lattice simulations

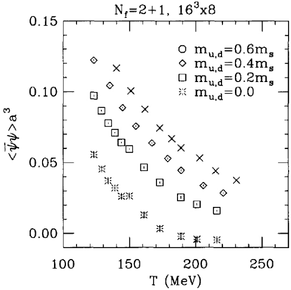


Figure: The chiral condensate $\langle \bar{\psi}\psi \rangle$ measured in lattice units in function of the temperature. The strange quark mass is fixed so that we have the physical value of the ϕ meson calculated on the lattice.

Lattice simulations

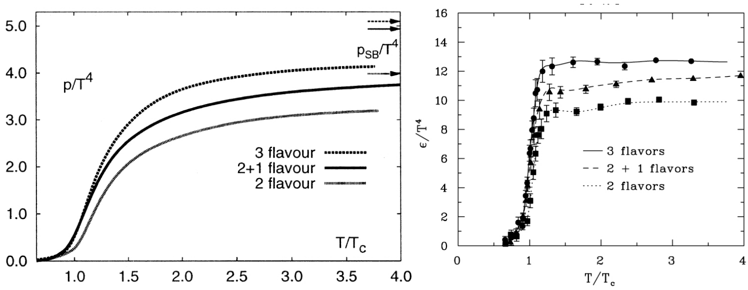


Figure: The pressure and the energy versus T/T_c for two, three flavour of light quarks and for two flavour of light and one of heavy quarks.

Lattice simulations in the physical point

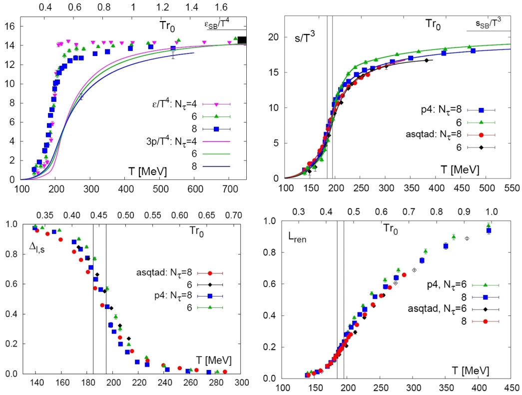


Figure: The pressure, the energy, the entropy, the chiral condensate and the Polyakov loop versus T/T_c in the physical point.

Lattice simulations in the physical point

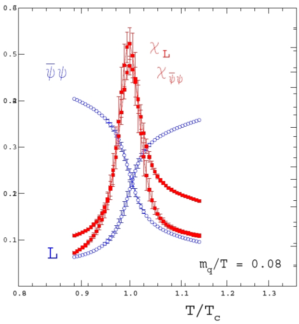


Figure: The chiral condensate $\langle\bar\psi\psi\rangle$ and the Polyakov loop vs. temperature in the physical point. The fact that the critical temperature is about the same for both transitions is not well understood yet.

Outline

Conclusion

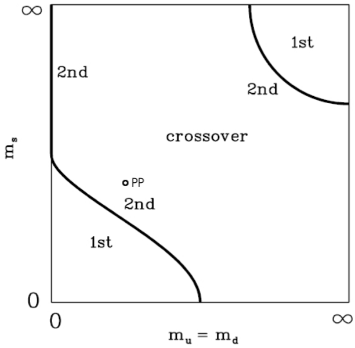


Figure: A possible phase diagram for QCD: Columbia Plot.

- Many thanks to Aleksi for his help.
- Thank you for the attention!