Goldstone's Theorem and Chiral Symmetry Breaking

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Outline

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- Basics of Classical Field Theory
- Symmetries and Noether's Theorem
- Transfer to Quantum Field Theory

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- Real, n-component Field ϕ.
- Dynamics: Lagrange Density $\mathcal{L}$.
- n Momentum Operators: $\pi^i(x) = \frac{\partial \mathcal{L}}{\partial(\partial_0 \phi_i(x))}$

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- Field Transformations:

$$\phi(x) \to \tilde{\phi}(x) = (R(\theta_1, \cdots, \theta_k))\phi(x)$$

- R = Representation of Lie Group:

$$R(\theta_1, \cdots, \theta_k) = e^{-i\theta_a\lambda_a} = 1 - i\theta_a\lambda_a + 0(\theta^2)$$

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Basics of Classical Field Theory

Variation of Lagrangian under Field Transformation:

$$\delta\mathcal{L}(\phi(x), \partial_\mu\phi(x)) = \mathcal{L}(\tilde{\phi}(x), \partial_\mu\tilde{\phi}(x)) - \mathcal{L}(\phi(x), \partial_\mu\phi(x))$$

(Use Euler-Lagrange)

$$= \theta_a \partial_\mu \left(-i \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \lambda_a \phi \right) = \theta_a \partial_\mu \left(J_a^\mu(x) \right)$$

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Symmetries and Noether's Theorem

- Symmetry: $\delta\mathcal{L}(\phi(x), \partial_\mu\phi(x)) = 0$
- $0 = \delta\mathcal{L} = \theta_a \partial_\mu (J_a^\mu(x))$
- Noether current:

$$J_a^\mu(x) = -i\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\lambda_a\phi \quad \text{with} \quad \partial_\mu J_a^\mu(x) = 0$$

- Noether charge:

$$Q_a(t) = \int J_a^0(t, \vec{x})\mathrm{d}^3x \quad \text{with} \quad \frac{\mathrm{d}}{\mathrm{d}t}Q_a(t) = 0.$$

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Transfer to Quantum Field Theory

- $\begin{pmatrix} \phi(x) \rightarrow \hat{\phi}(x) \\ \pi(x) \rightarrow \hat{\pi}(x) \end{pmatrix}$ + Canonical Commutation Relations
- Hilbert Space $\mathcal{H}$ of Physical States with Basis Vectors $|\alpha\rangle$.
- Current and Charge Operators: $\hat{J}^{\mu}_{a}(x)$ and $\hat{Q}_{a}(t)$
- For Symmetry Transformations: Noether's Theorem

$$\partial_\mu \hat{J}^{\mu}_{a}(x) = 0 \qquad \text{and} \qquad \frac{\mathrm{d}}{\mathrm{d}t}\hat{Q}_{a}(t) = [\hat{Q}_{a}(t), \hat{H}] = 0$$

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- Under Transformations: $\phi \to \tilde{\phi} \qquad |\alpha\rangle \to |\tilde{\alpha}\rangle$
- $\tilde{\phi} |\tilde{\alpha}\rangle = (\phi |\alpha\rangle)\tilde{} \quad \Rightarrow \quad |\tilde{\alpha}\rangle = e^{i\theta_a Q_a} |\alpha\rangle$
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- Behaviour of Lagrangian under Transformations.
- Variation of Lagrangian = Divergence of a Current.
- Noether's Theorem: Each Symmetry of a System gives rise to conserved Noether Currents and Noether Charges.
- Quantum Mechanics: Noether Currents and Noether Charges promoted to Operators.
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The QCD Lagrangian and its Symmetries

- 6 Quark Flavours (u,d,s,c,t,b)

$$\begin{pmatrix} m_u = 0.005\text{ GeV} \\ m_d = 0.009\text{ GeV} \\ m_s = 0.175\text{ GeV} \end{pmatrix} \ll 1\text{ GeV} < \begin{pmatrix} m_c \approx 1.2\text{ GeV} \\ m_b \approx 4.2\text{ GeV} \\ m_t \approx 174\text{ GeV} \end{pmatrix}$$

- 3 Colours (r,g,b)

$$q_f = \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix}$$

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The QCD Lagrangian and its Symmetries

- $\mathcal{L}_{QCD} = \sum_f \overline{q}_f (i\gamma^\mu \mathrm{D}_\mu - m_f) q_f - \frac{1}{4}\mathcal{G}_{a,\mu\nu}\mathcal{G}_a^{\mu\nu}$
- Covariant derivative: $\mathrm{D}_\mu = \partial_\mu - igA_\mu$

  is independent of Flavour.
- $\mathcal{G}_{a,\mu\nu}$: Components of Field Strength Tensor
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Chirality

- Chirality Matrix: $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$
- Projectors: $P_L = \frac{1}{2}(1-\gamma^5) \qquad P_R = \frac{1}{2}(1+\gamma^5)$
- Completeness, Orthogonality:
  $P_L + P_R = 1 \qquad P_L P_R = P_R P_L = 0$
- $q_{f,L} = P_L q_f \qquad q_{f,R} = P_R q_f \qquad q_f = q_{f,L} + q_{f,R}$

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The QCD Lagrangian and its Symmetries

Lagrangian in the Light Quark Sector:

$$\begin{aligned}
\mathcal{L} &= \sum_{f=u,d,s} \overline{q}_f (i\gamma^\mu \mathrm{D}\mu - m_f) q_f - \frac{1}{4}\mathcal{G}_{a,\mu\nu}\mathcal{G}_a^{\mu\nu} \\
&= \sum_{f=u,d,s} \{\overline{q}_{f,L}(i\gamma^\mu \mathrm{D}_\mu) q_{f,L} + \overline{q}_{f,R}(i\gamma^\mu \mathrm{D}_\mu) q_{f,R} \\
&\quad - m_f \overline{q}_{f,L} q_{f,R} - m_f \overline{q}_{f,R} q_{f,L}\} - \frac{1}{4}\mathcal{G}_{a,\mu\nu}\mathcal{G}_a^{\mu\nu}
\end{aligned}$$

The QCD Lagrangian and its Symmetries

$$\mathcal{L}^0 = \sum_{f=u,d,s}\{\overline{q}_{f,L}(i\gamma^\mu \mathrm{D}_\mu)q_{f,L} + \overline{q}_{f,R}(i\gamma^\mu \mathrm{D}_\mu)q_{f,R}\}$$

- Approximation: Massless Quarks.
- Consequence: Independent Left- and Right-Handed Fields.

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- Symmetry: $q_{f,L} \rightarrow e^{-i\theta_L} q_{f,L} \qquad q_{f,R} \rightarrow e^{-i\theta_R} q_{f,R}$
- Vector Transformations:

$$q_f \rightarrow e^{-i\theta_V} q_f \quad \text{Noether Current:} V^\mu = \overline{q}_f \gamma^\mu q$$

- Axial Transformations:

$$q_f \rightarrow e^{-i\theta_A \gamma^5} q_f \quad \text{Noether Current: } A^\mu = \overline{q}_f \gamma^\mu \gamma^5 q$$

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- Recall: Flavour Independence of Covariant Derivative
- $SU(3)_L^{Flavour} \times SU(3)_R^{Flavour}$ = Chiral Symmetry Group
- Vector/Axial Transformations:

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \rightarrow e^{-i\theta_V^b \frac{\lambda_b^F}{2}} \begin{pmatrix} u \\ d \\ s \end{pmatrix} \qquad q \rightarrow e^{-i\theta_A^b \frac{\lambda_b^F}{2}\gamma^5} q$$

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- $SU(3)_L^{Flavour} \times SU(3)_R^{Flavour}$ = Chiral Symmetry Group
- Vector/Axial Transformations:

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \rightarrow e^{-i\theta_V^b \frac{\lambda_b^F}{2}} \begin{pmatrix} u \\ d \\ s \end{pmatrix} \qquad q \rightarrow e^{-i\theta_A^b \frac{\lambda_b^F}{2}\gamma^5} q$$

Conserved Quantities

- Full Symmetry: $SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$
- 8+8+1+1 = 18 conserved Noether Currents.
- $V^\mu = \overline{q}_f \gamma^\mu q_f \quad A^\mu = \overline{q}_f \gamma^\mu \gamma^5 q_f$
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Chiral Symmetry Breaking

- So far: Massless Quarks

$$\begin{aligned}\mathcal{L}^M &= (\overline{q}_L + \overline{q}_R)M(q_f + q_R) \\ &= \begin{pmatrix} \overline{u}_L + \overline{u}_R \\ \overline{d}_L + \overline{d}_R \\ \overline{s}_L + \overline{s}_R \end{pmatrix} \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \begin{pmatrix} u_L + u_R \\ d_L + d_R \\ s_L + s_R \end{pmatrix}\end{aligned}$$

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- $\partial_\mu J^\mu = \delta \mathcal{L}$

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\partial_\mu V^\mu &= 0 \\
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- Massless Limit: Chiral Components are independent.
- Phase invariance: $U(1)_V \times U(1)_A$, 2 Noether Currents
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Outline

- Spontaneous Symmetry Breaking - An Intuitive Example
- Quantum Mechanical Proof

Outline

An Intuitive Example

- 2-Component, Real Field $\tilde{\phi} = (\tilde{\phi}_1, \tilde{\phi}_2)$

$$\mathcal{L} = \frac{1}{2}\partial_\nu \tilde{\phi}_1 \partial^\nu \tilde{\phi}_1 + \frac{1}{2}\partial_\nu \tilde{\phi}_2 \partial^\nu \tilde{\phi}_2 - \frac{\mu^2}{2}(\tilde{\phi}_1^2 + \tilde{\phi}_2^2) - \frac{\lambda}{4!}(\tilde{\phi}_1^2 + \tilde{\phi}_2^2)^2$$

- SO(2) Invariance.
- For $\mu^2 > 0$: Two Massive, Interacting Fields.
- For $\mu^2 < 0$: Minimum at $|\tilde{\phi}(x)| = v = \sqrt{-\frac{6\mu^2}{\lambda}}$.

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- Definition of SSB.
- Goldstone's Theorem: *In a theory that is spon. broken, each broken generator gives rise to a massless scalar particle.*
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$$\Phi_{ij} = \overline{q}_j q_i \quad \Pi_{ij} = i\overline{q}_j \gamma^5 q_i \qquad i, j \in \{u, d, s\}$$

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$$\pi_a(x) = \frac{1}{2}\mathrm{Tr}(\Pi(x)\lambda_a) = \frac{1}{2}\overline{q}(x)\lambda_a\gamma^5 q(x)$$

- Green function: $G^{\mu}_{ab} = \langle 0|\, T[A^{\mu}_a(x)\pi_b(y)]\,|0\rangle$
- $\frac{\partial}{\partial x^{\mu}} G^{\mu}_{ab} = \delta(x^0 - y^0)\,\langle 0|\,[A^{\mu}_a(x), \pi_b(y)]\,|0\rangle = -3i\delta^{(4)}(x-y)\delta_{ab}v.$
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- Consider only u,d Quark:

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- Negative Parity $\surd$
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- Assume: $m_u = m_d = m$ $\quad (SU(2)_{Vector}$ exact)
- $\partial_\mu A^\mu_b = i\bar{q}\{M, \sigma^F_b\}\gamma^5 q \approx 2m\pi_b$
- Assume: $\langle 0|\, T[\pi_a(x)\pi_b(y)]\, |0\rangle = C\delta_{ab} \int \frac{i}{p^2 - m_a^2} e^{-ip(x-y)} \mathrm{d}^4 p$

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