Maxwell‘s Demon

Reconciling Maxwell's demon with the second law based on the impossibility of information acquisition.

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Contents

Thermodynamics

- Conversion of heat into different forms of energy and vice versa
- Statistical predictions

We need statistical mechanics

(Kinetic Theory of Gases)

Kinetic Theory of Gases

- Small particles with mass
- Large number
- Random motion
- Elastic collisions with walls and each other

Three Laws

- First Law
  - Energy is conserved
  - $dU = \delta Q + \delta W$
- Second Law
  - Entropy of a closed system can only increase or stay constant
- Thrid Law

  $T \rightarrow 0$
  - All processes cease
  - Entropy approaches minimum value

Alternative formulations of the Second Law

Clausius:

Heat generally cannot spontanously flow from lower to higher temperature.

Alternative formulations of the Second Law

Kelvin:

It is impossible to convert heat completely into work in a cyclic process.

Alternative formulations of the Second Law

It is impossible for a closed system to produce any inequality of temperature, pressure or density without expenditure of work.

What is entropy?

Two more related ways of interpretation

- Macroscopic viewpoint
  - Classical thermodynamics
- Microscopic viewpoint
  - Statistical mechanics

Macroscopic viewpoint

- Differences tend to equalize over time
- Measure of how far equalization has progressed

Macroscopic viewpoint

$$dS = \frac{\delta Q_{rev}}{T} \text{ , for T = const } \Rightarrow \Delta S = \frac{\Delta Q_{rev}}{T}$$

and since $\delta Q_{rev} = dU + p\,dV$ (First Law)

$$\Rightarrow dS = \frac{dU}{T} + \frac{p}{T} dV$$

$\oint dS = 0$ for reversible cycles

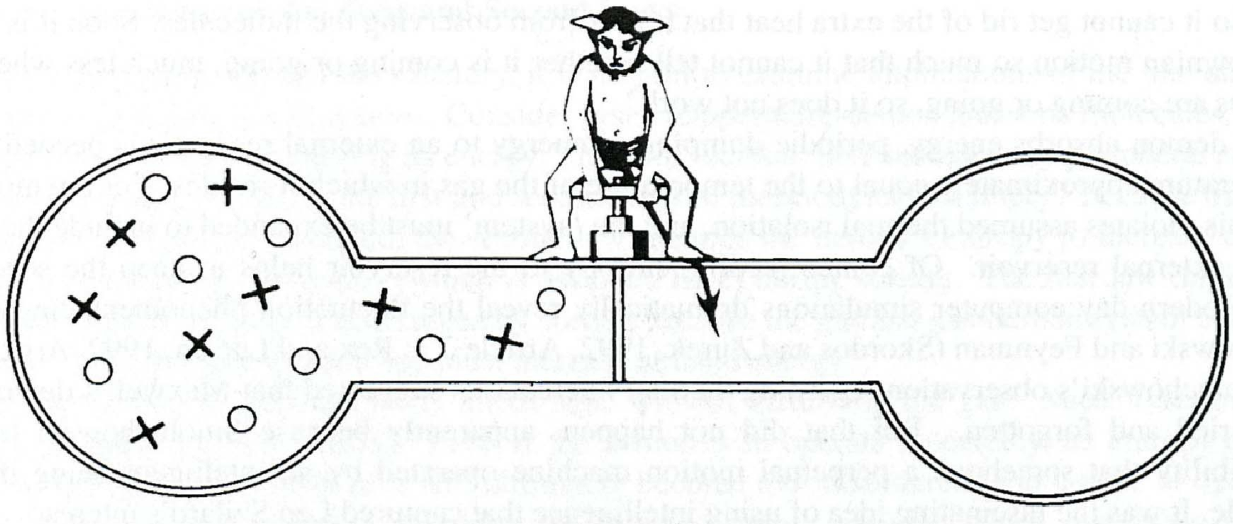
Microscopic viewpoint

- Amount of uncertainty, after observable macroscopic properties taken into account

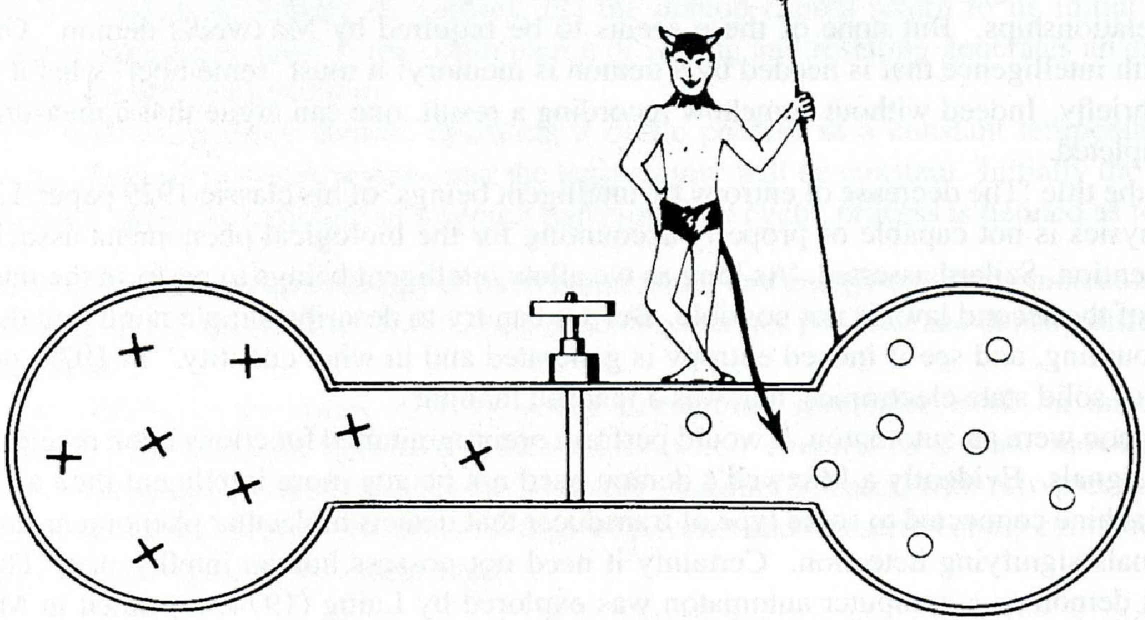
  $S = k_B \cdot \ln(P)$, P = # microstates corresponding to macrostate

- Equilibrium state maximizes entropy

  ~> all information about initial conditions lost

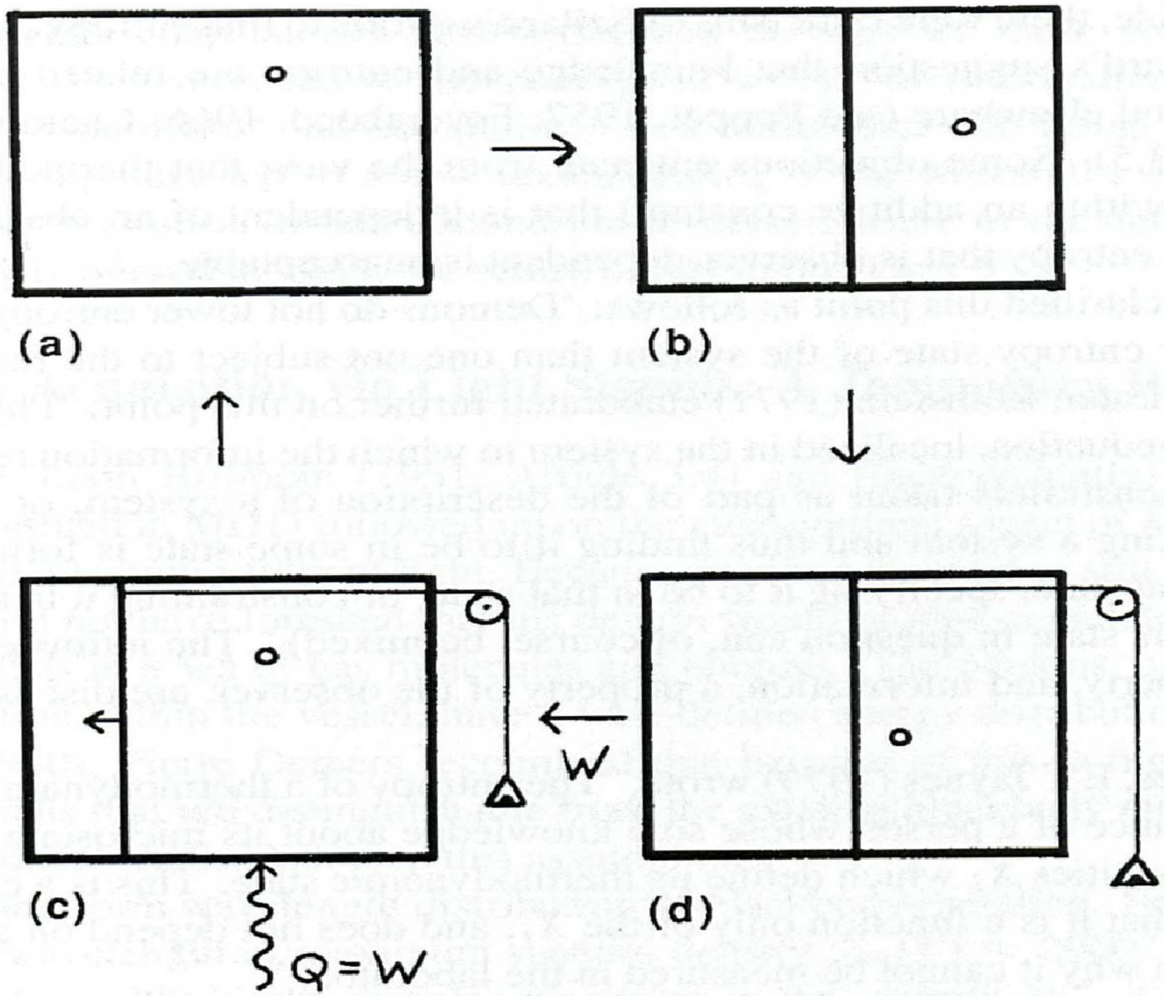


Maxwell's Demon



Maxwell's Demon: later

Szilard's model (1929)



Overall: Entropy of reservoir has been reduced and work has been done!

Szilard's model (1929)

$$e^{-S_1/k_B} + e^{-S_2/k_B} \leqslant 1 \qquad (*)$$

Assumption of the average produced entropy:

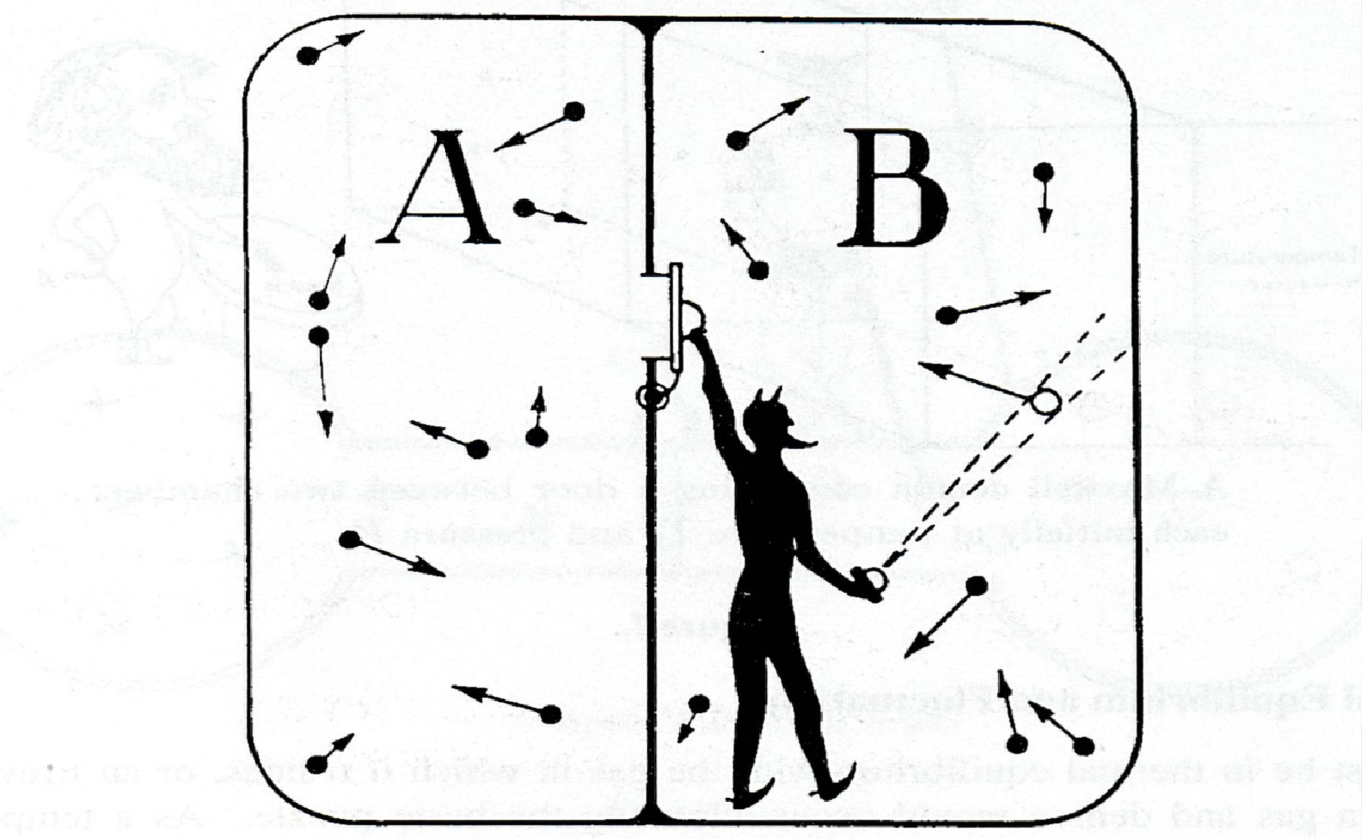
$$\bar{S}_1 = \bar{S}_2 = \bar{S} = k_B \cdot \ln(2)$$ (refer to Robin)

fulfills condition (*)

Conclusions on Szilard

- Szilard assumes the Second Law to be true
- Derives only a condition
- Assumes that the results of the measurement can be stored for a long time

Brillouin's model (1950)



- System in thermal equilibrium
  - Demon cannot see anything
  - Needs light source

Brillouin's model (1950)

Filament temperature $T_1 \gg T_0$

~> $h\nu_1 \gg k T_0$

Without demon:

$$S_{Bulp} = \frac{E}{T_1}$$

$$S_{Vessel} = \frac{E}{T_0} > S_{Bulp} > 0$$

~> global increase of entropy

Brillouin's model (1950)

With demon:

- At least one quantum of energy needs to be absorbed:

$$\Delta S_d = \frac{h\nu_1}{T_0} = k_B \cdot C \quad \Rightarrow \quad \frac{h\nu_1}{k_B T_0} = C \gg 1$$

- Use information to decrease entropy:

$$P_1 = P_0 - \delta p \quad , \delta p \ll P_0$$

$$\Delta S_i = S_1 - S_0 = k_B \cdot \ln\left(\frac{P_0}{P_1}\right) \approx -k_B \frac{\delta p}{P_0}$$

Brillouin's model (1950)

Overall entropy balance:

$$\Delta S = \Delta S_d + \Delta S_i = k_B \cdot \left(\underbrace{C}_{\gg 1} - \underbrace{\frac{\delta p}{P_0}}_{\ll 1} \right) > 0$$

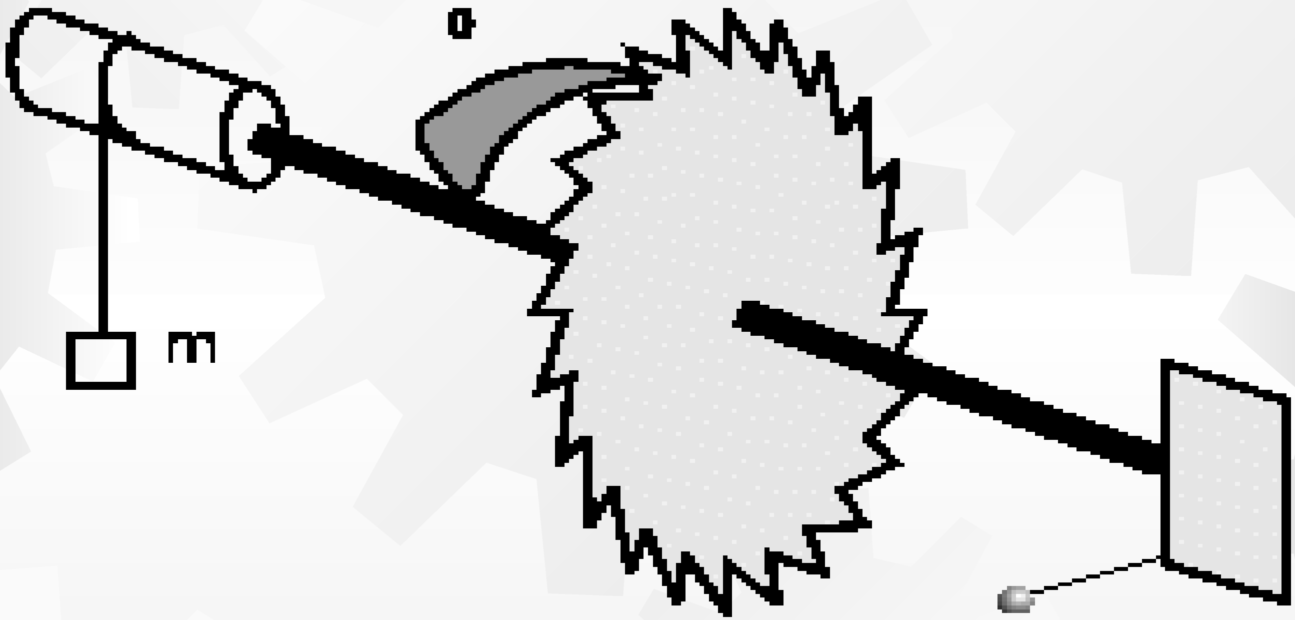
~> Second Law is saved

Observation in the laboratory

What is the smallest amount of entropy produced in an experiment?

- Read indications
- Light
- Power Supply
- etc.

Ratchet and pawl (Feynman)



Conclusions

- Performing measurements produces entropy
- 'Intelligent' beings have to be considered a part of the system
- Information can be regarded equivalent to (negative) entropy

Thank you for listening!

Questions?