ETH
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# Thermodynamik
# Serie 9

HS 07
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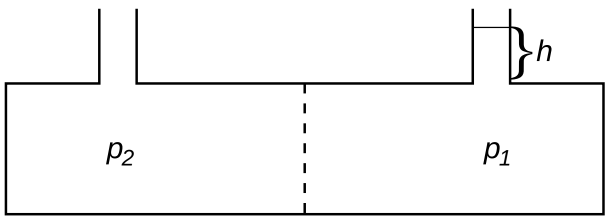
## Aufgabe 9.1 Entropy

From lecture notes (3.24) we have: $S = \frac{3}{2} N k_B \log T + N k_B \log V + S_0$.
From lecture notes (11.40) we have: $S = \frac{3}{2} N k_B \log T + N k_B \log \frac{V}{N} + S_0$.
Obviously (3.24) is false from two points of view:
a) $S$ isn't extensive.
b) $S$ leads to the Gibbs Paradox.
Show that with the substitution $V \rightarrow \frac{V}{N}$ both the problems are solved.

## Aufgabe 9.2 Osmotic pressure

Consider the following experimental arrangement for the measurement of the osmotic pressure:




A semipermeable wall (through which only the solvent can pass) separates the solvent (e.g. water, section 2, left hand side of the figure) from the concentrated solution (e.g. sugar in water, section 1, right hand side of the figure).
Derive again the expression for the osmotic pressure (lecture notes, page 94), using the chemical potential. Where is the higher column? Why?

## Aufgabe 9.3 Nucleation

Consider a $d$-dimensional elastic membrane (elasticity $C$) in $d+1$ dimensions, which can be deformed along the $d+1$th "transversal" direction (see figure). This transversal deformation can be described with $u(\vec{x})$, where $\vec{x} \in \mathbb{R}^d$. The deformation energy is $\propto (\nabla u)^2$ and the potential for the deformation is made up of a periodic part and of a coupling to a constant external force $F$:

$$V(u) = V_0 \left(1 - \cos(k_0 u)\right) - F u \, .$$

Then the free energy for the membrane is given by

$$H[u] = \int d^d x \, \left[ \frac{C}{2} (\nabla u)^2 + V(u) \right]$$

1. What is the potential as a function of external force? Which possibilities exist for the mobility of the membrane along the transverse direction? Define and determine a critical external force $F_c$.

2. Let's consider now $F \ll F_c$ and $d \geq 2$. Calculate the approximate energy $U(R)$ of a *nucleus* with radius $R$. This is a local deformation of the membrane, which disappears outside $R$ and is constant inside of $R$ with $u = u_0 = \frac{2\pi}{k_0}$. Make the assumption that the elastic and potential energy are of the same size and the deformation increases from 0 to $u_0$ with $R$ in a small area of width $w$ . Determine the critical radius R, above which the nucleus begins to grow. Why does $U(R_c)$ correspond to an activation energy, and how does it vary with $F$?

3. With $V_0 = 0$ the membrane moves according to $v = F/\eta$, with a speed $v$ and a friction coefficient $\eta$. Determine for our case with periodic potential and temperature $T > 0$, the drift velocity $v$ as a function of the force $F$; consider, in particular, the cases $F \to 0$ and $F \gg F_c$.

4. In one dimension (elastic line) $u_{\text{kink}}(x) = \frac{4}{k_0} \arctan(e^{x/w})$ with a constant $w$ is a minimum of the above energy function (free energy equation) for $F = 0$ which can represent a flank of the nucleus. The energy for a nucleus with radius $R$ and with such two flanks is then

$$U(R) \approx 2E_{\text{kink}}(1 - e^{-R/w}) - Fu_0R\,.$$

Calculate the critical radius $R_c$ and the activation energy. What is different if $F \to 0$ and what are the consequences for the speed of the line?

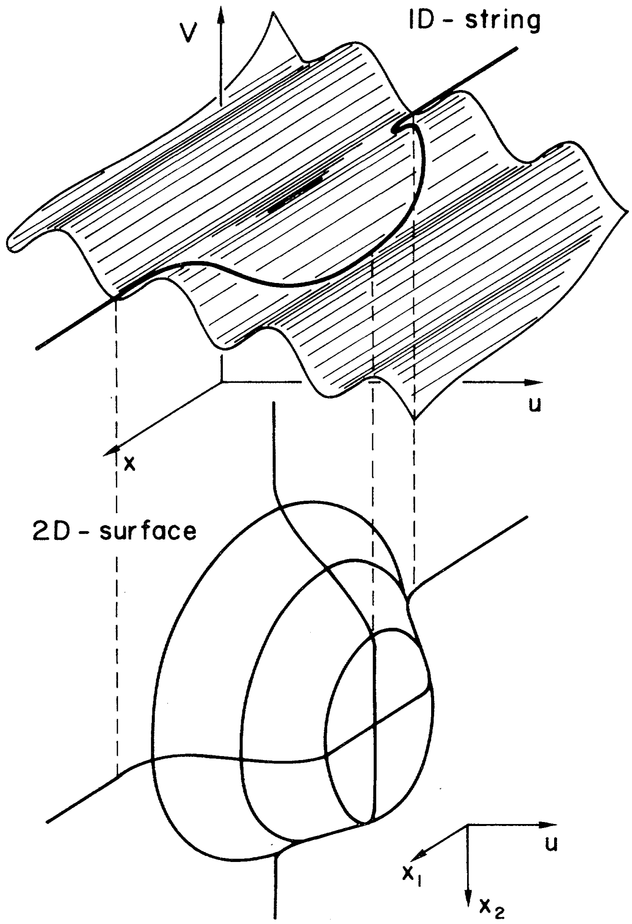



FIG. 13. Elastic manifold trapped in a (tilted) washboard potential. Top: One-dimensional elastic string with a finite segment (nucleus) activated to the next valley. The activation energy $2E_k$ involves the production of two kinks and remains always finite—the string is never in a "glassy" state. Bottom: Two-dimensional elastic surface with a finite nucleus activated to the next valley. The activation energy involves the creation of a one-dimensional (thin) wall, which costs an energy $2\pi r E_k$, where $r$ is the radius of the nucleus. If the nucleus is large enough, $r > r_c$, it expands and the elastic manifold moves on to the next valley. The critical radius $r_c$ increases with decreasing driving force $F$, $r_c = E_k / u_\circ F$, and the manifold shows glassy behavior with a diverging activation energy at vanishing driving force, $U(F) = \pi E_k^2 / u_\circ F$.

More background on the task in: G. Blatter *et al.*, Rev. Mod. Phys. **66** 1125, (1994).