ETH
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Statistical Physics
Exercise Sheet 2

HS 07
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Exercise 2.1 Thermodynamics of a magnetic system, part II

Consider a magnetic system whose differential work is given by $\delta W = H\,dM$. The thermodynamics of such a system is governed by the thermal and caloric equations of state: $U = U(T, M)$, $H = H(T, M)$.

(a) Show that

$$C_M - C_H = \left[H - \left(\frac{\partial U}{\partial M}\right)_T\right]\left(\frac{\partial M}{\partial T}\right)_H = T\left(\frac{\partial H}{\partial T}\right)_M \left(\frac{\partial M}{\partial T}\right)_H .$$

(b) Show that the internal energy U depends only on the temperature T if and only if the thermal equation of state is of the form $H = T\,f(M)$ (e.g. the ideal paramagnet of exercise 1.2). Compute $C_M - C_H$ in this case.

(c) Show that C_M depends only on the temperature T if and only if $H = g(M) + T\,f(M)$. Hint: Show first that

$$\left(\frac{\partial C_M}{\partial M}\right)_T = -T\left(\frac{\partial^2 H}{\partial T^2}\right)_M .$$

Exercise 2.2 The Linear Boltzmann equation

The subject of this exercise is a Boltzmann equation no longer emerging from two-particle scattering, as discussed in class, but from scattering of single particles by static impurities. We assume that the particles do not interact among themselves, but are influenced by a static background of impurities which causes a particle of momentum $\vec{p}$ to be scattered with new momentum $\vec{p}\,'$. We assume that the scattering happens within a negligibly short time interval and that it is elastic. Furthermore, we assume that each static scatterer is isotropic.

(a) Express these ideas mathematically: Find the relevant quantity describing the scattering, along with any of its symmetries, and write down the ensuing equation for $f(\vec{r}, \vec{p}, t)$.

(b) List all quantities φ conserved in the scattering, and prove the conservation law

$$\int d^3p\,\varphi(\vec{p})\left(\frac{\partial f}{\partial t}\right)_{\text{coll}}(\vec{r}, \vec{p}, t) = 0 .$$

(c) State and prove the "H-theorem" for the linear Boltzmann equation. When is H constant in time? How can this H-theorem be understood physically?