Theoretical Physics, Problem Set 10.

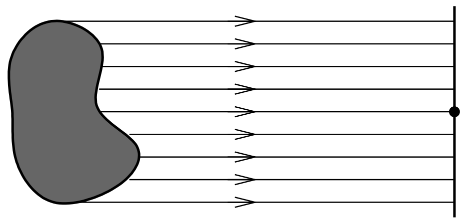
FS15 *Hand in: 6.05.15*

1. Seeing is not measuring

Descriptions of special relativity in popular science (e.g. Gamow 1940) claim that moving objects appear to be deformed; because of the Lorentz contraction in direction of the motion and the absence of the latter in the transversal direction. This is not true like that (Penrose, Terrell 1959; s. also Kraus and Borchers[1] 2005).

A measured length is not equal to a seen length. The measurement of length consists of the coordinate difference of two points of an object at the same time (w.r.t. a reference frame) (s. exercise 7.2(b)). Seeing however, is based on light signals which are received at the same time. The image thus combines earlier distant events with later closer ones. Though a moving object looks shorter than an identical object at rest at the same position in the image, it does not look distorted but rotated. That is, like a co-moving observer could see the object, if he had the right angle of view (compare aberration, exercise 8.2(c)). The latter would interpret the shortening simply as a perspective effect. The claim below allows us to understand the situation. Restrictively one has to say that (i) the above only holds for small scales: the proper transformations of the directions of sight is Möbius (s. exercise 8.1) and therefore angle preserving; moving objects which cover a large part of the sight field appear to be distorted; (ii) their colour and brightness is changed.

Consider an observer O who lets impact, at time $t = 0$, parallel light from an object onto a photographic plate which is perpendicular to it. (A camera which is focussed on a distant object gives the same picture up to scaling.)



A second observer O', with common origin $(t, \vec{x}) = (0, 0) = (t', \vec{x}\,')$ with O, but moved w.r.t. each other, does the same at time $t' = 0$. (Remark: due to aberration, the plates of O and O' are not parallel in general.) Claim: both get the same picture. In particular there is no length contraction.

For the derivation: the picture is generated by the simultaneous capture of side by side flying "light particles" (or at least one can imagine it like that). The claim thus follows from the following, to be justified statements:

Parallel inertial trajectories are mapped onto parallel inertial trajectories under Lorentz transformations. The property of light particles to fly side by side ($\Delta l = 0$ in the figure), is Lorentz-invariant. If it holds, then their spatial distance Δr is Lorentz-invariant.

[1] www3.interscience.wiley.com/cgi-bin/fulltext/109926451/PDFSTART

Hint: Two light particles P, Q may fly shifted. Consider two events $\mathcal{P}$, $\mathcal{Q}$, which happen to the particles P, Q respectively (e.g. to strike the plate). How does $(\mathcal{P} - \mathcal{Q}, \mathcal{P} - \mathcal{Q})$ depend on their time difference Δt?