Solid State Theory
Solution 11

Problem 11.1 Resistance of two barriers

The Landauer formula allows us to simply look for $|r/t|^2$ of the entire system and then multiply it by $R_k = \pi\hbar/e^2$. We start with the transfer matrix relating the wavefunction in an area n of the material to that in an area $n+1$ via $\psi_{n+1} = T\psi_n$ is:

$$T = \begin{pmatrix} \frac{1}{t^*} & -\frac{r^*}{t^*} \\ -\frac{r}{t} & \frac{1}{t} \end{pmatrix}$$

If we have two barriers then we simply have:

$$\psi_1 = T_1\psi_0$$
$$\psi_2 = T_2\psi_1$$
$$\psi_2 = T_2T_1\psi_0$$

We take the product of the diagonal of $T = T_2T_1$ to obtain an expression for $1/|t|^2$ which we then rearrange using the fact that $|t|^2 + |r|^2 = 1$ to obtain:

$$\frac{|r^2|}{|t^2|} = \frac{1}{|t^2|} - 1 = \frac{1 + |r_1^2||r_2^2| - |t_1^2||t_2^2|}{|t_1^2||t_2^2|} + \frac{r_1 r_2^*}{t_1^2|t_2^2|} + \frac{r_2 r_1^*}{t_2^2|t_1^2|}$$

We now average over relative positions, meaning the two last terms pick up a phase factor and vanish while the first one does not.

$$\frac{|r^2|}{|t^2|} = \frac{1 + |r_1^2||r_2^2| - |t_1^2||t_2^2|}{|t_1^2||t_2^2|}$$

We then again use $|t|^2 + |r|^2 = 1$ to obtain:

$$\frac{|r^2|}{|t^2|} = \frac{|r_1^2| + |r_2^2|}{|t_1^2||t_2^2|} = \frac{|r_1^2|}{|t_1^2|}\left(1 + \frac{|r_2^2|}{|t_2^2|}\right) + \frac{|r_2^2|}{|t_2^2|}\left(1 + \frac{|r_1^2|}{|t_1^2|}\right)$$

Plugging this in to the Landauer formula then gives us the final result:

$$R = R_1 + R_2 + 2\frac{R_1R_2}{R_k}$$

Here we see the increased resistivity due to the second barrier leading to localisation.