Solid State Theory
Exercise 6

Problem 6.1 Specific Heat of a Semiconductor and Graphene

In Lecture 6 we calculated the specific heat for a metal. Now we compare it to that of a semiconductor and graphene. The specific heat at constant volume is defined as

$$c_V = \frac{1}{V}\left(\frac{\partial U}{\partial T}\right)_{V,N} \tag{1}$$

where U is the internal energy of the system.

a) Calculate the specific heat of an undoped semiconductor under the assumption $k_\mathrm{B}T \ll E_g$, where E_g is the band gap. Show that it contains an ideal gas-like contribution $(3/2)n(T)k_B$ where $n(T)$ is the number of excitations, and a correction. Is this correction small or large?

   To proceed, use the parabolic approximation for the band spectrum with effective mass. The chemical potential $\mu(T)$ has to be calculated from the condition, that the number of electrons in the conduction band ($n_e(T)$) is equal to the number of holes in the valence band ($n_h(T)$).

b) Calculate the specific heat of graphene at half filling. Note that the perfect particle-hole symmetry fixes chemical potential to the Dirac nodes at all temperatures. To simplify the calculation, approximate the dispersion around the two Dirac points as

$$\varepsilon_\mathbf{k} = \pm\hbar v_\mathrm{F}|\mathbf{k}|, \tag{2}$$

   where $\mathbf{k}$ is relative to the position of a Dirac node.

Problem 6.2 Spin Susceptibility of a Metal, a Semiconductor, and Graphene

The Pauli spin susceptibility is defined as

$$\chi_\mathrm{Pauli} = \left(\left.\frac{\partial M}{\partial H}\right|_{H=0}\right)_{T,V,N} \tag{3}$$

where M is the net electron magnetization.

a) Calculate the Pauli spin susceptibility of a metal due to its conduction electrons. Assume that the magnetic field couples only to the electron spin.

b) Calculate the spin susceptibility of a semiconductor with gap $E_g > 0$ and compare the result to an ideal paramagnetic gas.

c) Calculate the spin susceptibility of graphene

Use the same assumptions about the electron spectra as in Exercise 1.