**Exercise 1.** ***Green's functions for the wave equation***

a) Prove, without using Fourier transforms, the identity

$$\int_{-\infty}^{\infty} d\omega \, e^{-i\omega x} = 2\pi\delta(x). \tag{1}$$

The Green's function for the d' Alembert operator can be written in the following form

$$G(\mathbf{x},t) = \lim_{\delta_1,\delta_2\to 0} c \int \frac{d^3k dE}{(2\pi)^4} \frac{-e^{-i(cEt-\mathbf{k}\cdot\mathbf{x})}}{2|\mathbf{k}|} \left[\frac{1}{E-|\mathbf{k}|+i\delta_1} - \frac{1}{E+|\mathbf{k}|+i\delta_2}\right]. \tag{2}$$

The result depends on the choice of sign for the regulators $\delta_1$ and $\delta_2$.

For $\delta_1 > 0$ and $\delta_2 > 0$ the result is the *retarded* Green's function

$$G_{ret}(\mathbf{x},t) = \frac{1}{4\pi|\mathbf{x}|}\Theta(t>0)\,\delta\left(t - \frac{|\mathbf{x}|}{c}\right). \tag{3}$$

b) Show that for $\delta_1 < 0$ and $\delta_2 < 0$ the Green's function is the *advanced* Green's function

$$G_{adv}(\mathbf{x},t) = \frac{1}{4\pi|\mathbf{x}|}\Theta(t<0)\,\delta\left(t + \frac{|\mathbf{x}|}{c}\right). \tag{4}$$

c) Calculate the Green's function for the remaining two cases ($\delta_1 > 0$ and $\delta_2 < 0$ and vice versa). You should find that only the sum of the two can be brought to a similar form as the cases above.

**Exercise 2.** ***Electric Dipole Radiation***

Imagine two tiny metal spheres at distance $d$ from each other connected by a wire (see Figure 1), where at time $t$, the upper sphere carries a charge $q(t) = q_0\cos(\omega t)$ while the charge on the lower sphere is given by $-q(t)$.

a) Calculate the electric potential far away from the dipole. Use $d \ll r$ and $d \ll \frac{c}{\omega}$.

b) Take the limit of $\omega \to 0$. What do you expect?

c) Now look at the case where also $r \gg \frac{c}{\omega}$, that is, when we are interested in large distances from the source in comparison to the wavelength ($r \gg \lambda$). How does the expression for the potential simplify in this case?

d) Obtain an expression for the vector potential in the limit $d \ll r$ and $d \ll \frac{c}{\omega}$.

e) Calculate the resulting electric and magnetic fields in the same limit with also $r \gg \frac{c}{\omega}$ .

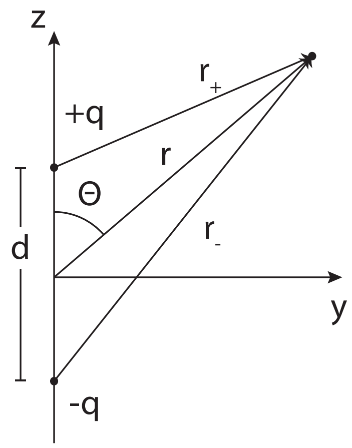


Figure 1: Electric Dipole

**Exercise 3.** ***Spherical waves***

Find the direction of the electric field $\vec{E}$ and the magnetic field $\vec{B}$ of a spherical wave, with respect to the direction of propagation.

*Hint. Use Maxwell equations.*